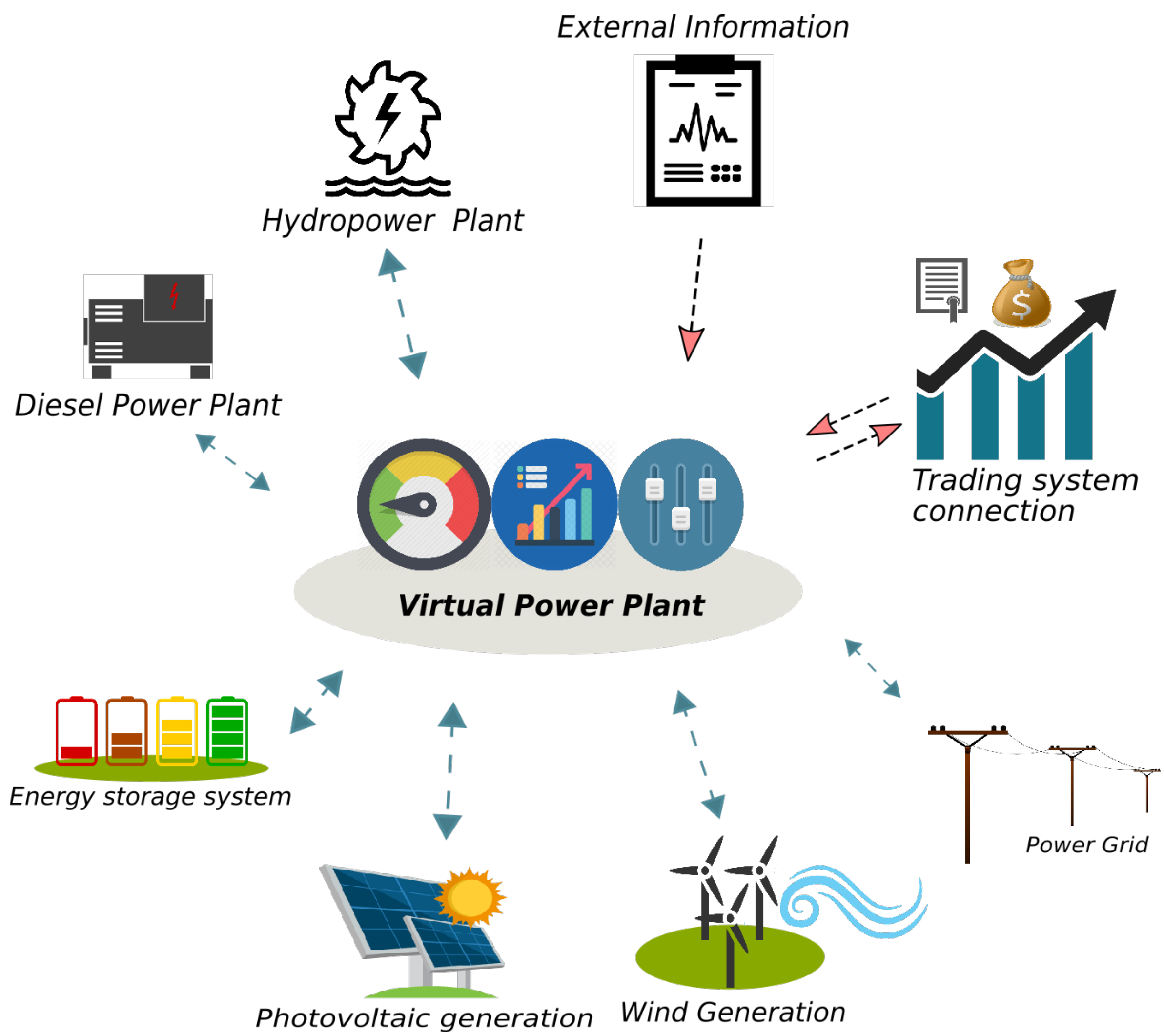
Optimal Operation of Multiple Microgrids and Distributed Resources Under The Concept Of Virtual Power Plant Using Convex Optimization



Ivan Dario Valencia Hincapie

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Dissertation submited to the deparment of Electrical Engineering of the Universidad Tecnológica de Pereira in partial fullfilment of the requeriments for the degree of
Master in Electrical Engineering

Pereira, July 2018
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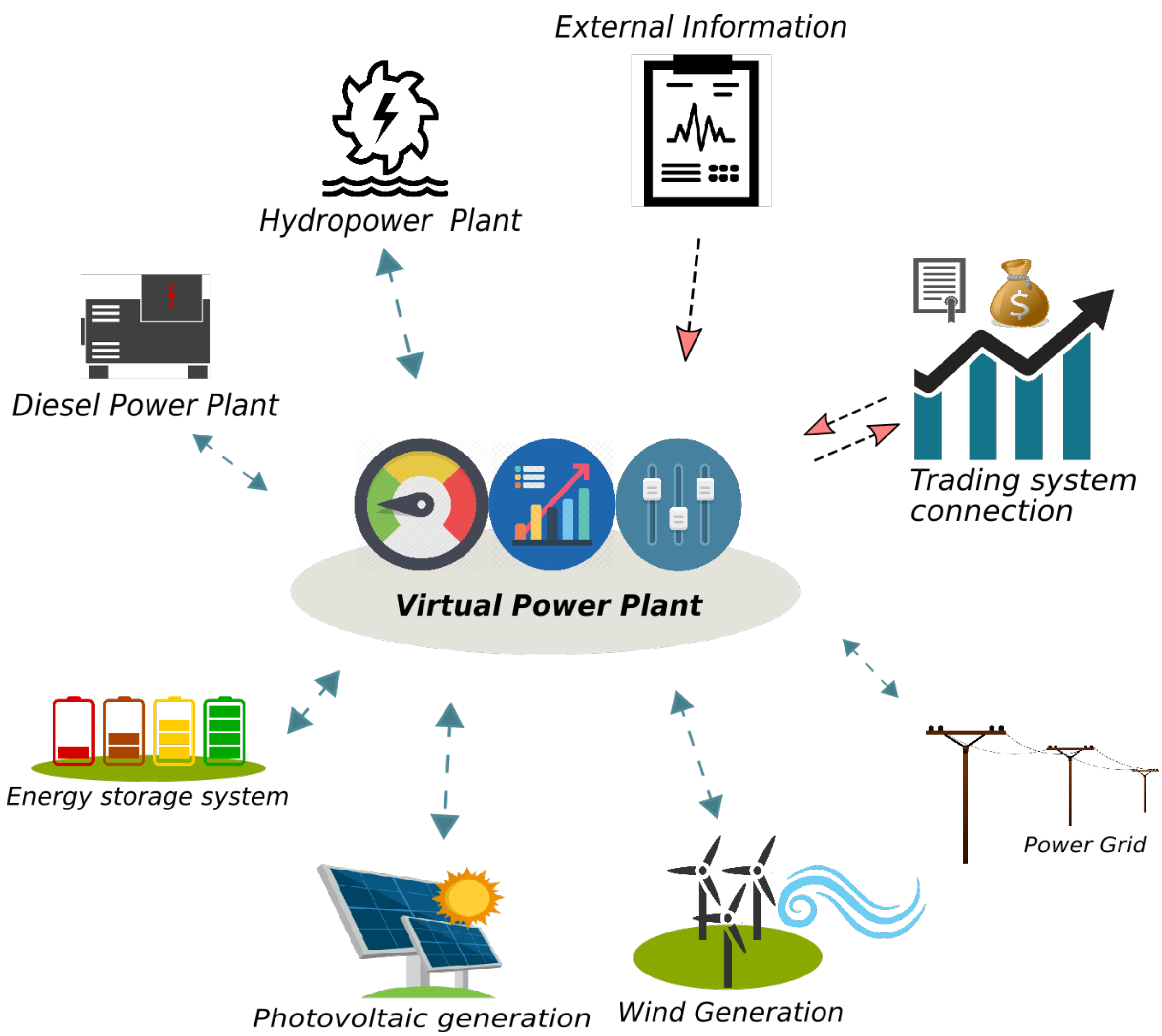

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Abstract

The transition of electrical power system from hierarchical systems characterized by a top-down approach with large and centralized power plants, to one more environmental-friendly electrical system characterized by the use of distributed energy resources (DER), requires an effort to prevent technical problems and reduce the commercial and regulatory barriers. In this context, the concept of virtual power plant (VPP) emerges as an alternative to support the high penetration of DERs reducing the technical problems and giving visibility in the energy market. In this way, all DER are managed in a coordinated fashion through a virtual power plant that can participate in market as a conventional power plant.

The operation of a VPP requires the treatment of random variables as solar radiance or energy market prices to perform an optimal economic dispatch. Similarly in real time operation, the VPP needs to take the optimal decisions to prevent penalization for deviation from scheduled values.

In this document it is proposed the control and operation of a VPP with convex optimization where the economical dispatch problem including uncertainties in solar radiance and market prices is modeled as a convex optimization problem, and for the real time operation it is used a convex model to solve the problem of preventing penalization for deviation from scheduled power, while the technical and physical constraints of power flow are fulfilled. It is used the second-order cone programming to performs a convex relaxation of the non-linear and non-convex power flow equations.

The proposed VPP is tested in 3 scenarios with DER penetration of 10%, 40% and 70% over an IEEE test system. The results demonstrate the accuracy of the proposed approximation and the viability of the proposed methodology.

Resumen

La transición del sistema eléctrico de un sistema jerárquico con grandes plantas eléctricas centralizadas a un sistema más amigable con el medio ambiente caracterizado por el uso de recursos de energía distribuidos, requiere un esfuerzo para prevenir los problemas técnicos y reducir las barreras comerciales y de regulación. En este contexto el concepto de planta de potencia virtual (VPP por sus siglas en inglés) emerge como una alternativa para soportar la alta inclusión de recursos de energía distribuidos (DER por sus siglas en inglés) reduciendo los problemas técnicos y otorgado visibilidad en el mercado eléctrico a los propietarios de los DER, de esta forma los DER son agrupados a través de una planta de potencia que participa en el mercado como una planta convencional.

La operación de la VPP requiere el manejo de variables aleatorias como la radiación solar o el precio de la energía para realizar un despacho económico óptimo, de forma similar en la operación en tiempo real la VPP requiere tomar decisiones óptimas para prevenir penalizaciones por el desvío de la potencia ofertada.

En este documento se propone el control y la operación de una VPP usando optimización convexa donde el problema de despacho económico incluyendo la aleatoriedad en la radiación solar y precios del mercado es modelado como un problema de optimización convexa, y para la operación en tiempo real se usa un modelo convexo que resuelve el problema de prevenir la penalización por el desvío de la potencia programada mientras se cumple con las restricciones física y técnicas del flujo de potencia. Mediante programación cónica de segundo orden se realiza una relajación convexa de las ecuaciones no lineales y no convexas del flujo de potencia.

La VPP propuesta es probada en 3 escenarios donde se simula una inclusión del 10%. 40% y 70% de los DER sobre un sistema de pruebas de la IEEE. Los resultados demuestran la precisión de la aproximación propuesta y la viabilidad de la metodología.

Contents

Chapter 1

Introduction

1.1 Problem description

The electric sector has a high responsibility in the global warming due to the greenhouse-gas emissions during the combustion of fossil fuels, such as coal, oil and natural gas to produce electricity in conventional power plants [1] [2]. Recent alternatives to produce environmental-friendly energy have drawn attention over distributed energy resources(DER) like energy storage systems, distributed generators and controllable loads.

The high penetration of DER into the power grid is still problematic, since DERs are largely invisible to the grid given their small capacity and power fluctuations. Once a DER is individually integrated into the grid, the resulting problems may relate to technical, commercial, and regulatory barriers.

One way to support integration of DER are through micro-grids where distributed generators are close to demands and can operate connected to the main grid or in island mode. Grid-connected microgrids can have, in some moments, surplus or deficit of energy. Therefore, a coordinated approach is required.

Micro-grids and independent distributed generation has the same problem, they have small capacities and their power production depends on uncertain variables such as solar radiance or wind speed which create unexpected power fluctuations. Due to previous features, they are weak in front of electricity market where they should pay a punishment in the case of deviation from their scheduled productions making their participation very risky.

In this context, the concept of virtual power plant (VPP) play a significant role since it makes possible integration of multiple micro-grids with distributed generation, energy storage and controllable loads; all these working coordinately in order to behave like a conventional power plant. The VPP can decide to participate on the day-ahead market to maximize its profit, with role of producer or consumer based on the direction of exchanged power and offer technical services to the grid.

However, operating a VPP with high penetration of distributed energy resources and multiple microgrids for economical benefits requires dealing with the uncertainties in renewable energy resources to get a well defined day-ahead schedule as well as maintain the scheduled production taking optimal decisions in *real time* to prevent penalization in case of production deviations.

Real time operation imply times from 1 to 5 seconds. Therefore, the mathematical techniques must be fast enough and guarantee optimality since *it is a control problem*. Convex optimization fulfills these requirements. However, the original problem could be non-linear and non-convex with a high degree of complexity.

Taking into account all of the above, the following research question is formulated:

¿How to integrate DER and multiple microgrids under a virtual power plant using convex optimization for optimal real time operation?

1.2 Justification

The environmental crisis derived by greenhouse gas emissions have drawn attention over electricity sector in which carbon dioxide (CO_2) makes up the vast majority of greenhouse gas emissions. Other gases like methane (CH_4) and nitrous oxide (N_20) are also emitted but in smaller amounts. These gases are released during the combustion of fossil fuels, such as coal, oil, and natural gas, to produce electricity in conventional power plants (CPP)[1]. Large hidropower also contributes to the emission of CH_4 to the atmosphere (see Figure 1.1)

This kind of generation is related historically with electric systems characterized by a top-down approach where large and centralized power plants dominated electricity generation[3].

However, the system is shifting away from this traditional approach in the direction of distributed generation (DG) which have caught widespread attention because of their renewable, clean, and flexible characteristics to produce environmental friendly energy.

The increase share of uncoordinated activity of distributed energy resources (DER) over the grid creates serious problems for system operators. Nevertheless, with the accelerated development of technology and smart devices it is possible to improve the control over electric network. New generalized concepts under *smart grid* term are explored to reduce global warming and to make more intelligent use of distributed energy resources. VPP is one of the most promising technologies in the context of smart grids.

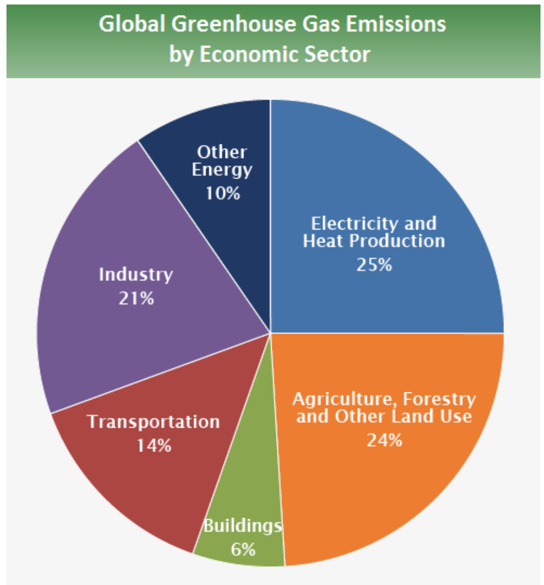

Figure 1.1: Global greenhouse emissions by economic sector. image taken from [1]

Nowadays, this research area has been investigated by the European Union through different projects, in especial the Fenix project that support massive penetration of Distributed Energy Resources (DER) through aggregation into Large Scale Virtual Power Plants (LSVPP)[4]. In Denmark, it was proposed a VPP to control electric vehicles contributing to regulating power reserves.

In Colombia there is an strategic route for the transition to smart grids compiled in studies developed by UPME (*unidad de planeación minero energetica*) together with IDB (Interamerican Development Bank), Ministry of mines and energy and Ministry of TICs which was named *Smart Grids, Colombia vision 2030*[5]. This study demonstrated that the main challenges for the smart grid in Colombia are related to power distribution systems. Therefore, micrograds and VPP are key technologies for development of the smart grid in the Colombian context.

1.3 Objectives

1.3.1 General

To develop a convex optimization model for real time optimal operation of multiple microgrids with high penetration of distributed energy resources (e.g renewable energies and energy storage) in order to create a virtual power plant. This model will consider variable energy market-prices and an approximated model of the grid.

1.3.2 Specific

- To achieve the state of the art of virtual power plants and the Colombian context about it.
- To define a convex model for day-ahead scheduled of a market-base virtual power plant.
- To define a convex model for optimal operation of VPP given grid technical constraints.
- To perform tests to validate the effectiveness of the proposed optimization model.
- To analyze the results in the Colombian context.

1.4 Research framework

1.4.1 General definitions

As aforementioned, the evolution of electric power systems across the world has been characterized by generating electricity by large synchronous generators and transmit through power lines which connect geographically dispersed loads. Over the past decades there has been an increasing requirement for more electricity generation in order to meet the growing demand[6]. The transition from a conventional to an environmentally friendly system has drawn attention over distributed energy resources (DER)[7]. DER describes many different type of technologies, including distributed generation (DG), energy storage systems and controllable loads. Each one of these terms is described below except controllable loads which exceeds the scope of this project; the concept of microgrid is also explained.

1. Distributed Generation (DG)

   DG units tend to be geographically spread over large areas and have relatively low generating capacities. Increasing the number of generation stations and distributing them over a larger geographic area decreases transmission distances, reduces energy loss and lowers the pressure on the grid. DG are used to either supplant or supplement centralize power in order to meet local demand efficiently[8]. There are several DG technologies that can be considered for integration into VPP or microgrid, the more used are: natural gas generator, diesel generator, biomass and bio-gas, combined heat and power (CHP), and wind power and solar farms.

2. Energy Storage System (ESS)

   Energy storage (ES) plays a crucial role in managing power balance and stability, by absorbing energy from the grid when it is in excess and then supplying it back to the grid when required[9]. There are many different types of electrical ES technologies, they can be divided into electromagnetic, mechanical and electro-chemical energy systems[10] (see Figure 1.2).

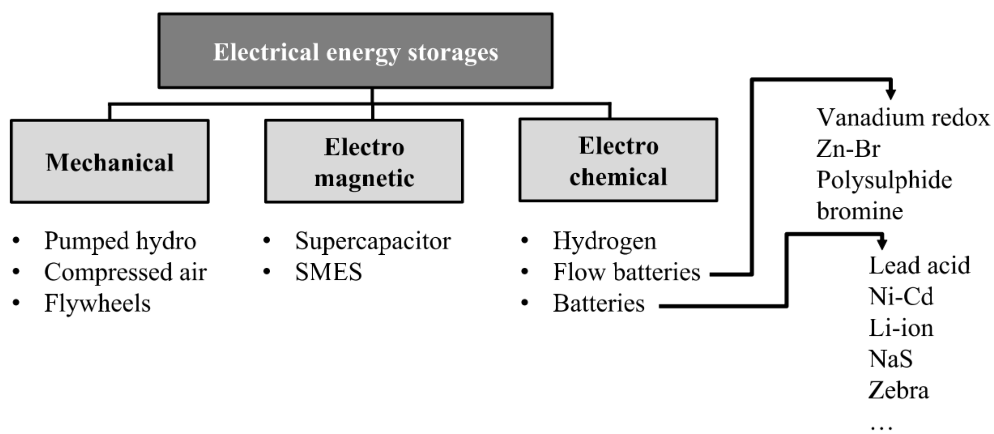

Figure 1.2: Electrical ES. Image taken from [10]

   Some important technologies that can be considered for integration of VPP are listed: battery energy storage system, flywheel energy storage system, supercapacitor energy storage, hydrogen along with fuel cell and hydraulic pumped energy storage system.

3. Microgrids

   Microgrid can be defined as an integrated energy system consisting of distributed energy resources and multiple electrical loads operating as a single autonomous grid with the characteristic of operate either in parallel to or *islanded* from power grid. (see Figure 1.3)

   The main feature of microgrid is its self-sufficiency, which means that it can maintain self-control, safety and management. It acts as a complete separate

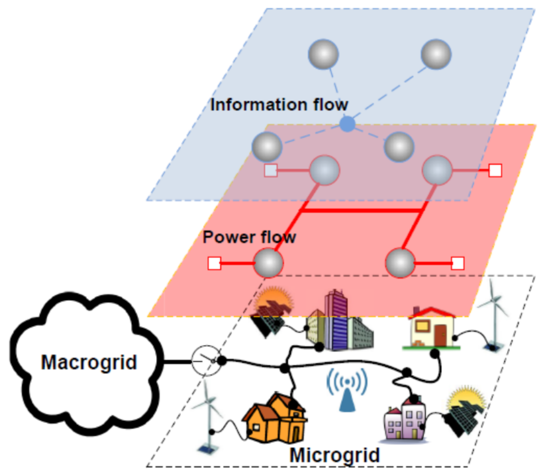

Figure 1.3: Organization of microgrid. Image taken from [11]

power system for achieving power balance control, system operation optimization, protection and fault detection, power quality control function, etc. [12]

1.4.2 Convex optimization

Convex optimization is a special class of mathematical optimization problems that has been studied for about a century, but in recent years has taken importance in areas such as automatic control system, estimation and signal processing, communications and networks, data analysis and modeling, electrical engineering, etc. Steve Boyd in his book **convex optimization** says [13]:

> The advantages of formulating a problem as convex optimization problem is that the problem can be solved very reliably and efficiently, using interior-point methods or other special methods for convex optimizations. These solution methods are reliable enough to be embedded in a computer-aided design or analysis tool, or even a real-time reactive or automatic control system.

Formulating a problem as convex optimization problem has the advantage of get unique an optimal solution and in particular case of VPP has the characteristic of the problem can be solved in *real time* which is a special condition for control and operation of a VPP.

1.5 Contributions

The main contribution of this work are:

- A convex optimization model of economical dispatch of VPP that include variation in energy market prices and solar radiance.
- A convex optimization model for real time operation and control of VPP that include the physical and technical constraints of grid using second-order cone optimization to describe the power flow constraints.
- Implementation in python of the proposed models for operation of the VPP using the cvxpy library.

Is important to note that the proposed VPP is for the distribution system in the Colombian context and is a little step for incursion of DER since at the moment there are not VPPs in Colombia; on the other hand the use of convex optimization to model electrical problems has been little studied in the national context.

1.6 Document outline

The structure of this document is organized as follow, first in Chapter 1 a brief description of the context and the main objective of this work is done, next more detailed information about the state of the art of VPP is described in Chapter 2, in Chapter 3 is shown the mathematical models of VPP for economic dispatch and optimal operation in its natural form (deterministic, non-convex, non-linear) and in Chapter 4 the convex representation of models presented in previous chapter are explained. Finally in Chapter 5 it is exposed the results of the proposed VPP in three different scenarios and in Chapter 6 it is written the conclusions, recommendations and future works.

Chapter 2

Virtual Power Plant

2.1 General concepts

2.1.1 Concept of VPP

The concept of VPP has its origins in the book "*The Virtual Utility: Accounting, Technology and Competitive aspects of the Emerging Industry*"[14] where Doctor Shimon Awerbuch in 1997 defined the Virtual Utility as:

> The Virtual Utility (VU) is a flexible collaboration of independent, market-driven entities that provide efficient energy service demanded by consumers without necessarily owning the corresponding assets.

Similar to the Virtual Utility concept which purposes to use emerging technologies to provide customer-oriented energy service, the idea of the VPP is to aggregate different types of DER through an advanced information and communication technology infrastructure for the better use of those available resources, this cluster of DER working coordinately through the VPP results in a behavior close to a large conventional power plant, offering technical and commercial advantages leading to cost reductions and improving controllability in order to offer and buy power when electricity market and environmental conditions are optimal. This can be considered a generalization of VPP concept despite there is no an exact definition. (see Figure 2.1)

2.1.2 Classification of VPPs

The study and implementation of VPPs in different contexts have allowed classify them into different categories[15][16], namely:

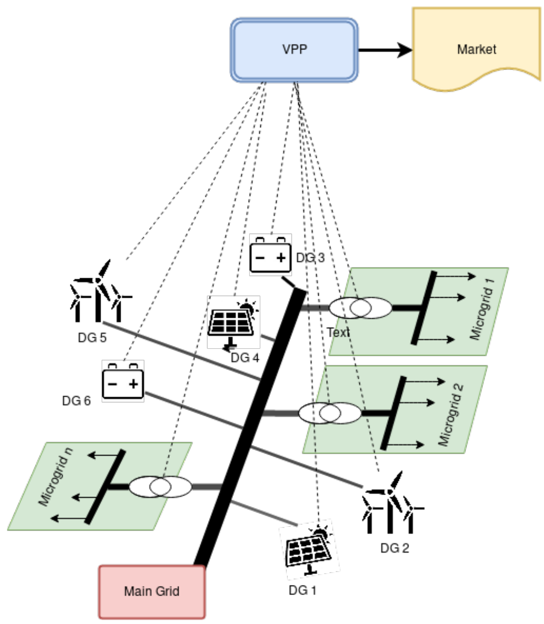

Figure 2.1: Concept of VPP.

1. VPP according participation in power system

   VPPs are classified by their role in energy market, more specific the purpose of DER aggregation, and can be classified into two categories: Commercial VPP (CVPP) and Technical VPP (TVPP), in [17] a CVPP was developed to participate in the day-ahead (DA) electricity market where the main goal of VPP is the maximization of the DA profit in conjunction with the minimization of the anticipated real-time production, similarly in [18] and [19] where DER are aggregated to participate in DA electricity market.

   On the other hand TVPP works at the transmission boundary level so that it is visible to the system operator. Information from DER in the local network is passed by few CVPPs to TVPP. This facilitates distribution system operator handle the operating set points, parameters and cost data from each DER in the network with detailed information about topology and constraint[20].

2. VPP according geographical scale

   Another point having in mind is the geographical scale of DER aggregation. The European project CRISP define the aggregation of different VPP spread over the network as *Large Scale VPP* (LSVPP)[21]; similarly, in [22], it is proposed an integration of VPP by levels. First level are local VPPs (LVPP); these are grouped through a second level or regional virtual power plant (RVPP) which is used by distribution system operator and markets. Finally, RVPPs are grouped in a LSVPP, which main goal is to serve the TSO requirements.

3. VPP control arquitectures

   In [23] the following virtual power plant categorizations are developed:

   - Centralized Controlled VPP (CCVPP): In this case the VPP needs to have the complete knowledge of associated DER. This kind of control can reach more easily the optimal operation, but with the inconvenient that is hard to scale.
   - Decentralized Controlled VPP (DCVPP): DCVPP is based on a hierarchical architecture with several distributed local controllers and a central controller in a high level that guarantees the system operation security and global economical benefits.
   - Fully Decentralized Controlled VPP (FDCVPP): This is an extension of DCVPP, with the difference that central controller is replaced by information exchange agents which only provide valuable data (market price signal, weather forecasting and data logging etc). This control architecture has a relatively higher scalability as it relies on plug and play ability of DER aggregation.

4. Portafolio of DER

   Taking into account that the VPP is compose by DG, storage and flexible loads, this can be divide by the type of DERs presented in [12]:

   - First type is *supply-side* VPPs comprising DG units.
   - Second type is *Demand-Response* which consist of combination of flexible loads and storage
   - Third type is *mixed asset* having both previous features in it

2.1.3 VPP's strategic advantages

The flexible aggregation of distributed energy resources have caught widespread atention from differents stakeholders by its benefits, DER owners, distribution system operators, transmission system operator and end-consumers are benefited by the virtual power plant services some of them are the following:

- **Accessibility**: Through the VPP concept is possible to provide access to the market to individual DER owners who cannot reach minimum level entrance market constraints, offering as alternative to join them as a single resource. Also the VPP can handle with the high penetration of electric vehicles and use it as energy storage system and/or controllable loads offering Vehicle to Grid service (V2G).

- **Reliability**: With a large number of DER the VPP improves the capacity to deal any outages in the system and on the other hand technical services like demand response, voltage control and frequency control improve the reliability of the system.

- **Optimality**: With different types of distributed energy resources the VPP can optimize the planning and the operation involving technical constraints with the aim of reduce losses in the grid.

- **Competitivity**: It is a good alternative to support the high penetration of DER which leads to more services in the market and in the grid, this is important because the cost energy for end-consumers can be reduced.

- **Profitability**: The optimal operation of a VPP can maximize the profit of DER owners and customers of the VPP can benefit from its services.

2.1.4 VPP's implementations

Several projects about VPP have been developed in the last decade, the Fenix project [4] is a demonstration of LSVPP in Europe which participate in the day-ahead electricity market and provide tertiary reserve, voltage control services and solve network contingencies. Virtual power plants based on the idea of power matcher with decentralized market-based control are proposed in [24] [25]. In [26] a VPP was developed to control electric vehicles, and in 2016 the company AGL launched a big solar virtual power plant in South Africa, providing 5MW of peaking capacity and offering to customers the opportunity to save on their energy bills[27].

2.2 The Colombian case

In the Colombian context the efforts for thinking the transition of electrical system to one more smart, interconnected and friendly with the environment are reflected in the document *Plan energético nacional Colombia: ideario energético 2050* created by

Unidad de Planeación Minero Energética (UPME) in 2015[28] and in 2016 a joint work done between the Universidad Tecnológica de Pereira, Ministrerio de tecnologías de información y comunicación, UPME, Inter-American Development Bank, Ministerio de minas y energía and the initiative "Colombia inteligente" named *Smart Grids Colombia visión 2030*[29] where the path to develop the transition of electrical system was proposed according to the following strategic objectives that can be summarized as:

- *Universal Access*: There are several non-interconnected areas in Colombia where is important to guarantee the supply of energy through DER and smart grids, and thus potentiate the economic and social development of these regions.
- *Safety and quality*: With the high penetration of distributed energy resources and latest technology it seeks to achieve a more efficient electric system that reduce the risk of power outages to guaranteeing a reliable and efficient energy supply.
- *Competitiveness*: The implementation of the optimal and efficient energy system that support integration of renewable energies and new information technologies will improve country competitiveness
- *Sustainability*: With the incursion of renewable energies and the efficient management of DER the environmental impact will be reduced.

With these documents as support it is expected that the change of electric system can be made progressively. It requires efforts in the development or adaptation of new technologies and changes to the legislation and regulation of new players in the electric market. In legal terms the Colombian government promulgated the law 1715 in 2014 which aims to promote the development of non-conventional sources of energy, with a particular emphasis on those of a renewable nature. This law gives different tax benefits for using non-conventional energy sources (FNCE by its acronym in Spanish). After that in 2015 decree 2143 was issued as the regulatory framework of law 1715 of 2014 establishing the necessary requirements for DER owners to access the tax benefits enunciated in law 1715.

One of the main challenges for implementation of the VPP concept is the participation in spot market with differential prices. This work assume that. Although, at this moment there is no VPP example operating in Colombia.

2.3 Proposed Virtual Power Plant

In this document it is proposed a virtual power plant for the distribution system that is composed by microgrids, diesel plants, photo-voltaic generation and batteries as energy storage system (see Figure 2.2). The aim of this VPP is to participate in the energy market with variable market-prices through an economic dispatch for day-ahead increasing profit of DER owners and DSO.

Since the VPP operates in the distribution system, the owner is the distribution system operator (DSO) which signs bilateral contracts with DER owners and it is benefited by the VPP technical services. In this way the DSO can support the high penetration of distributed energy resources maintaining the system stability and preferring DER power over main grid power (slack) by its lower costs.

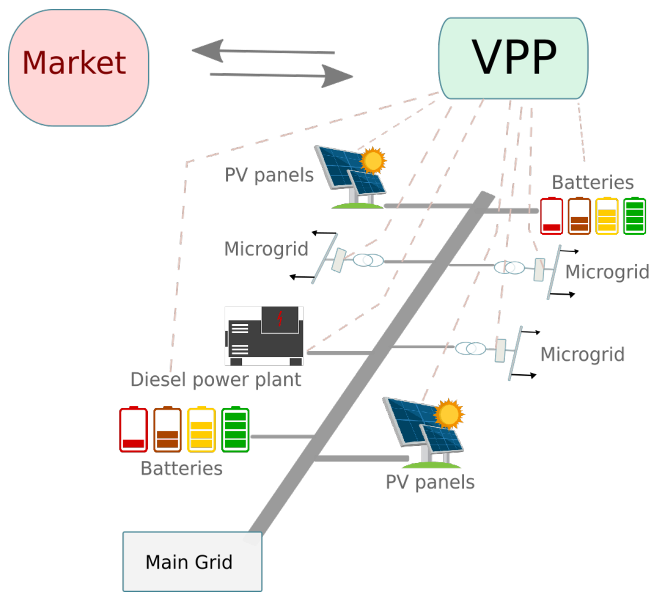

Figure 2.2: Scheme of proposed VPP.

For a better management of microgrids it is supposed that the microgrid are integrated through the *aggregator* concept that has been proposed in [30], [31] y [32] for electric vehicles (EV) integration. Aggregator can been seen as an interface between the VPP and microgrids doing all operational activities required by the VPP (see Figure 2.3). The aggregator for example can collect historical data of microgrid and process it through an artificial neural network that gives the power prediction and send it to VPP for real time operation.

Operation of VPP requires obviously of an advanced information and communication technology infrastructure that is possible now with evolution in power electronics, communications and software.

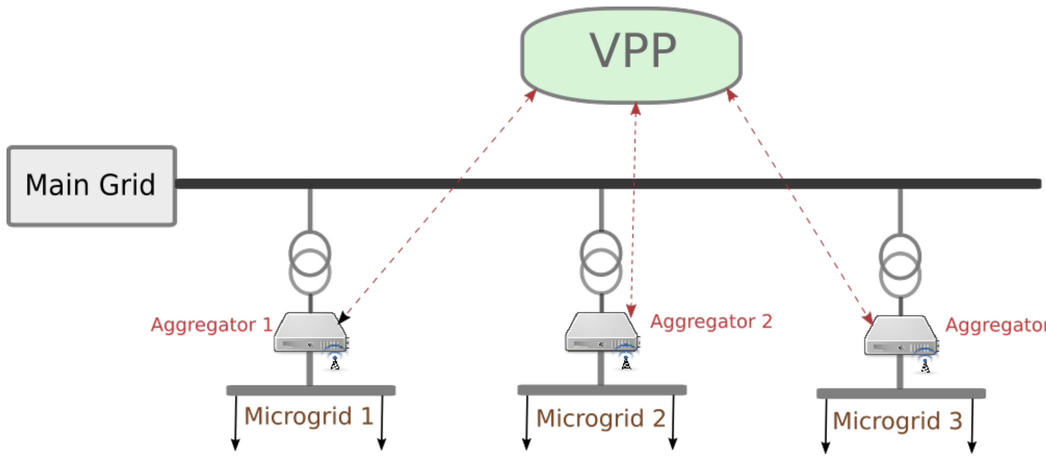

Figure 2.3: Aggregator concept for microgrid in VPP

Taking into account the aforementioned, the rest of the document is build over the following VPP assumptions:

1. The operator of the VPP is the distribution system operator
2. The owner of batteries storage system is the VPP operator
3. The VPP operator sells/buys to the spot market with differentials prices
4. The DER owner can sign bilateral contract with the VPP operator (DSO)
5. Each distributed resource are integrated by power electronic converter
6. Microgrids are integrated through aggregators
7. It is possible to obtain prediction of solar radiance and market price in a time horizon
8. It is possible to obtain microgrids demand/power prediction in a time horizon

Chapter 3

VPP operation and mathematical models

The optimal operation of the VPP is divided in two stages as depicted in Figure 3.1. In the first stage, the economic dispatch problem is solved using the forecasting of the spot prices, and the expected solar radiation. This problem is called day-ahead and presents a simplified model which neglects the constraints related to the grid. The result of this problem gives the optimal bidding of the VPP in the spot market. It is important to notice that under the assumptions of this work (see Section 2.3), there is a subroutine that makes accurate prediction of prices and solar radiation. This subroutine is beyond the scope of this work, although general features include fast and accurate predictions for practical implementation.

In the second stage, the technical constraints of the grid are included. In this case, the objective is to redistributed the resources in order to avoid penalization for deviations of the day-ahead dispatch. A receding horizon approach is proposed for taking optimal decisions based in short term predicted data for real time operation.

3.1 Economic dispatch of VPP

The problem of determining the optimal offering strategy of virtual power plant in day-ahead market can be modeled as a maximization problem of the VPP profit where the power generation cost is subtracted from the power sold in the market.

The VPP are composed by the set of diesel generation $\mathcal{D}$, the photovoltaic generation $\mathcal{PV}$ and batteries $\mathcal{B}$ with cardinality Nd, Npv, Ns respectively. The diesel generation cost is defined as:

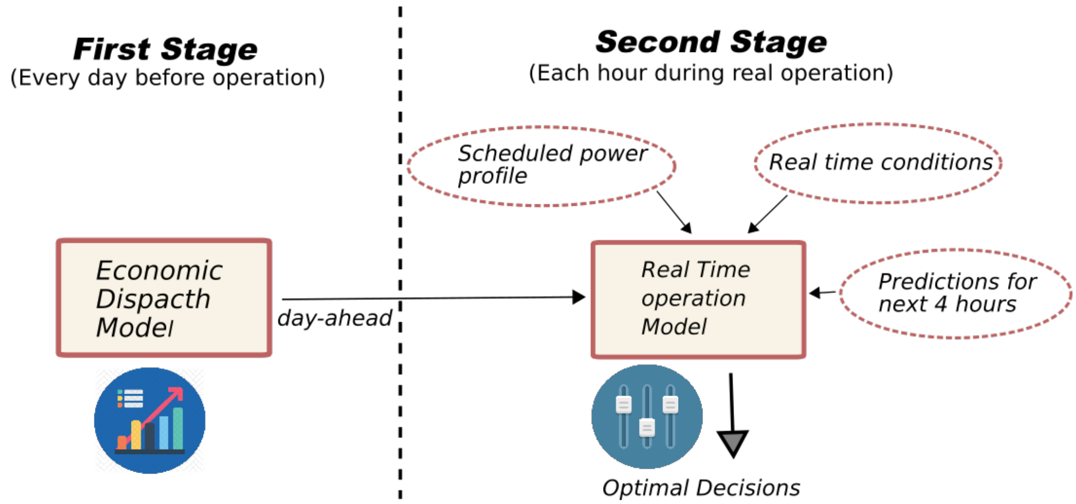

Figure 3.1: Stages for optimal operation of VPP

$$dieselGenCost_{(t)} = C_{lt\_d} \cdot \sum_{d=1}^{Nd} \left(\alpha_d P_{d(t)}^2 + \beta_d P_{d(t)} \right) \quad P_d \in \mathcal{D} \tag{3.1}$$

Where C_{lt_d} is the cost of diesel liter, Nd is the amount of diesel generators, $P_{d(t)}$ is the power output of unit d at time t and α_d, β_d are coefficients cost of unit d, so the economic dispatch problem is defined as follow:

Model 1 *Deterministic DAY-AHEAD-MODEL*

$$\max_{\psi_t} \sum_{t=1}^{24} \left[C_{m(t)} \cdot P_{vpp(t)} - C_{DSO_d} \cdot dieselGenCost_{(t)} \right] \tag{3.2}$$

Subject to

$$\sum_{d=1}^{Nd} P_{d(t)} + \sum_{s=1}^{Ns} P_{s(t)} + \sum_{pv=1}^{Npv} P_{pv(t)} = P_{vpp(t)} \qquad P_d \in \mathcal{D}, P_s \in \mathcal{B}, P_{pv} \in \mathcal{PV} \tag{3.3}$$

$$P_{s(t)} = \frac{\Delta E_{s(t)}}{\Delta t} \qquad P_s \in \mathcal{B}, \Delta t = 1 \tag{3.4}$$

$$|P_{s(t)}| \leq P_{s\_max} \qquad P_s \in \mathcal{B} \tag{3.5}$$

$$\Delta E_{s(t)} = E_{s(t-1)} - E_{s(t)} \tag{3.6}$$

$$E_{s(1)} = E_{s(24)} = E_{fixed} \tag{3.7}$$

$$0 \leq E_{s(t)} \leq E_{s\_max} \tag{3.8}$$

$$0 \leq P_{d(t)} \leq P_{d\_max} \quad P_d \in \mathcal{D} \tag{3.9}$$

Where set $\psi_t = \{P_{vpp(t)}, P_{d(t)}, P_{pv(t)}, P_{s(t)}, E_s, \Delta E_{s(t)}\}$ is composed by the optimization variables of problem corresponding to VPP power, diesel power, photovoltaic power, batteries power, batteries energy and energy variation in batteries respectively. Notice that the model is a simple linear programming model.

Equation 3.2 is the objective function that reflects the maximization of profit, generating the optimal dispatch of all elements in VPP for each hour of day (t). Two main terms compose the objective function:

1. $\mathrm{C_{m(t)}} \cdot \mathrm{P_{vpp(t)}}$ describes the incomes achieved if power is sold or cost incurred if power is bought in VPP for its participation in day-ahead market.

2. $C_{DSO\_d} \cdot dieselGenCost_{(t)}$ describes the costs of VPP power generation from diesel power plants where C_{DSO_d} is the price that VPP pays to diesel generator owner. that parameters are fixed by VPP owner.

Constraints of this model are the following:

- *The power balance*: Constraint 3.3 is an equality constraint which means that the total VPP's power is equal to the sum of its DER power

- *The Battery power*: Constraint 3.4 is defined as the energy variation at time t over Δt where Δt is considered as 1 hour.

- *Energy in batteries*: Constraint 3.6 define the energy variation in batteries and 3.7 impose that the energy stored at the last time period must be at least equal to the energy stored at the beginning of the planning horizon

- *Power limits*: Constraints 3.5, 3.8, and 3.9 define the limits of the power on batteries, energy on batteries and diesel power respectively

3.1.1 Characteristics of the economic dispatch in the VPP

It is important to note that the deterministic day-ahead math model correspond to a convex optimization problem with a quadratic objective function. Another thing to have in mind in the economic dispatch problem is the dealing with unexpected values of renewable resources like solar radiance, and by the side of market, dealing with fluctuation in market-prices. These uncertainties are not included in Model 1 because depending of the way that they are included the final model can be convex or non-convex optimization problem. The way in which uncertainties are handled is explained in the next chapter.

3.2 Real time operation

The real time operation of VPP involves the solution to power flow equations with the aim to maintain the power system under technical constraints as physical laws and operational restrictions, while demand at all buses are satisfied with minimal generation cost preventing penalization for deviation from scheduled power profile.

3.2.1 Optimal operation of the VPP

Since the VPP operates in the distribution system it is considered the power network $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ where $\mathcal{N}$ denotes the set of nodes (or buses) and $\mathcal{L}$ denotes the set of distribution lines, DERs are connected to a subset of nodes denoted as $\mathcal{K} \subseteq \mathcal{N}$, it is supposed that there is electric demand and generation in each node. The nodal admittance matrix is defined as $\mathrm{Y} \in \mathbb{C}^{|\mathcal{N}|\times|\mathcal{N}|}$ which has components $\mathrm{Y_{ij}} = \mathrm{G_{ij}} + \mathrm{iB_{ij}}$ for each line $(i, j) \in \mathcal{L}$, and $G_{ii} = g_{ii} - \sum_{i\neq j} G_{ij}, \quad B_{ii} = b_{ii} - \sum_{i\neq j} B_{ij}$ where $\mathrm{g_{ii}}$ is shunt conductance and $\mathrm{b_{ii}}$ is susceptance at bus $i \in \mathcal{N}$. Let $\mathrm{P_i^g}$, $\mathrm{Q_i^g}$, $\mathrm{P_i^d}$ and $\mathrm{Q_i^d}$ be the real and reactive power output and load respectively at bus i. The complex voltage V_i at bus i is defined in the rectangular form as $V_i = e_i + if_i$ where $|V_i|^2 = e_i^2 + f_i^2$ is the voltage magnitude.

With the previous notation and based on B.Kocuk's work in [33] the optimal operation of the VPP can be formulated as:

Model 2 *Optimal operation of the VPP.*

$$\min \sum_{i\in\mathcal{K}} C_i(P_i^g) \tag{3.10}$$

Subject to:

$$P_i^g - P_i^d = G_{ii}(e_i^2 + f_i^2) + \sum_{j \in \delta(i)} [G_{ij}(e_i e_j + f_i f_j) - B_{ij}(e_i f_j - e_j f_i)] \quad i \in \mathcal{N} \tag{3.11}$$

$$Q_i^g - Q_i^d = -B_{ii}(e_i^2 + f_i^2) + \sum_{j \in \delta(i)} [-B_{ij}(e_i e_j + f_i f_j) - G_{ij}(e_i f_j - e_j f_i)] \quad i \in \mathcal{N} \tag{3.12}$$

$$\underline{V}_i^2 \leq e_i^2 + f_i^2 \leq \bar{V}_i^2 \quad i \in \mathcal{N} \tag{3.13}$$

$$P_i^{min} \leq P_i^g \leq P_i^{max} \quad i \in \mathcal{K} \tag{3.14}$$

$$Q_i^{min} \leq Q_i^g \leq Q_i^{max} \quad i \in \mathcal{K} \tag{3.15}$$

In this model the objective function $C_i(p_i^g)$ is typically linear or convex quadratic in the real power output P_i^g of generator i, the Constraints (3.11) and (3.12) correspond to the conservation of active and reactive power flows at each node. δ(i) denotes the set of neighbor nodes of node i. Constraint (3.13) guarantees the technical restriction of voltage magnitude at each node. Finally in (3.14) and (3.15) the limits of active and reactive power output at each generator are fixed.

It is important to note that the optimal operation definition (2) is a non-convex and non-linear optimization problem where non-convexity and non-linearity come form Constraints (3.11), (3.12) and (3.13). That characteristic of the problem make it a NP-hard problem that is difficult to solve.

Chapter 4

VPP operation with convex optimization

4.1 Convex optimization

Convex optimization is a special class of mathematical optimization problems where convexity characteristic of problem guarantee a unique solution that can be reached in polynomial time, in other words the computational effort it takes to solve a problem instance to within a prescribed error tolerance is in the worst case proportional to a polynomial in the number of variables and constraints. That is an advantage for control problems like the VPP operation where unique and optimal solution can be reached fast.

4.1.1 Definitions

Before presenting the application of convex theory in the VPP model let us define some basic concepts.

Definition 1 *(Convex set): a set $C \subseteq \mathbb{R}^n$ is convex provided that, for any $x, y \in C$ and $\lambda \in [0, 1]$ we have*

$$\lambda x + (1 - \lambda)y \in C$$

i.e, the line segment joining x, y lies entirely in C (see Figure 4.1)

Definition 2 *(Convex function): a function $f : \mathbb{R}^n \to \mathbb{R}$ is convex if its domain is a convex set and for all x, y in its domain, and all $\lambda \in [0, 1]$ we have:*

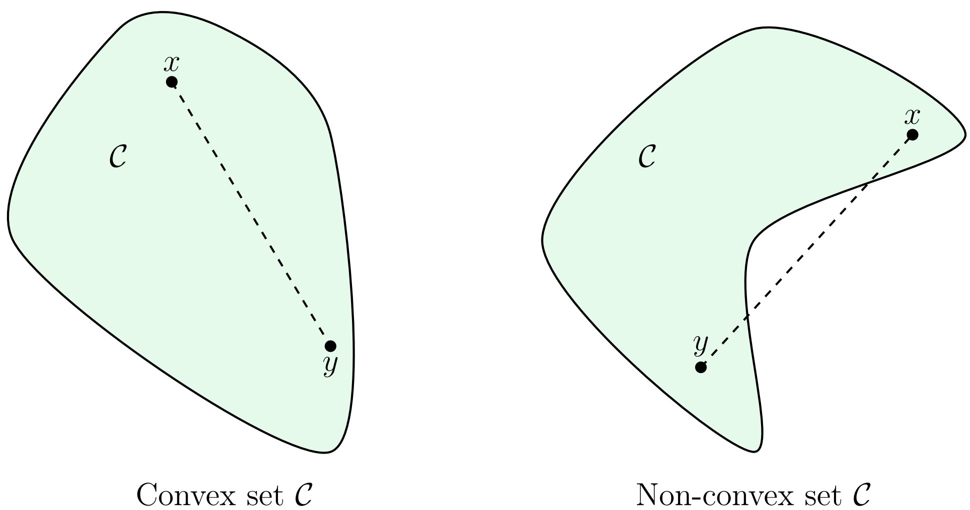

Figure 4.1: Example of convex and non-convex set

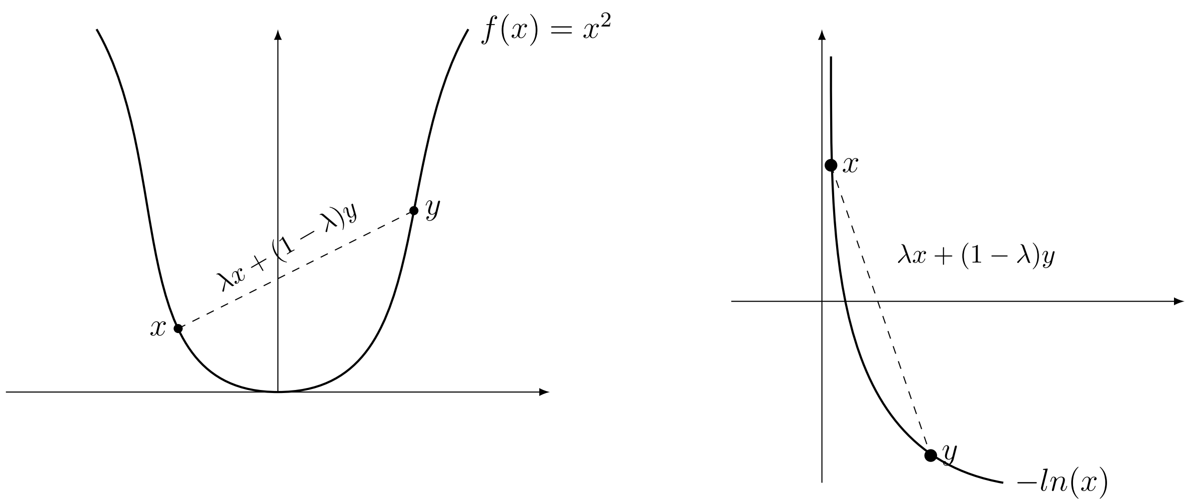

Figure 4.2: Example of two convex functions

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y)$$

Figure 4.2 shows an example of two convex functions.

Definition 3 *(Convex problem): a convex optimization problem is one of the form:*

$$\begin{array}{rll} \text{minimize} & f_0(x) & \\ \text{subject to} & f_i(x) \leq b_i, & i = 1, ..., m \end{array}$$

Where the functions $f_0, ..., f_m : \mathbb{R}^n \rightarrow \mathbb{R}$ are convex. Which means that Definition 2 is satisfied for all $x, y \in \mathbb{R}^n$ and $\lambda \in [0, 1]$

One of the main features of convex optimization comes from the following theorem taken from Nesterov work [34]:

Theorem 1 *let f be a strongly convex function on $\mathbb{R}^n$ and Q be a closed convex set. Then there exists a unique solution x^* of the following problem:*

$$\text{minimize}\{f(x) \mid x \in Q\}$$

Theorem 1 also indicates that in a convex optimization problem all local optimum is a global optimum and if the objective function is strongly convex (as in the case of this work), then the solution is unique. The proof of this theorem can be seen in [34].

Convex optimization problems also have practical advantages since it can be solved very reliable and efficiently, using interior-point methods or other special methods for convex optimization. These solution methods are reliable enough to be embedded in a computer-aided design or analysis tool, or even a real-time reactive or automatic control system.

4.1.2 Second-order Cone Programming (SOCP)

Second-order cone programming (SOCP) problems are a subset of convex optimization problems that include linear and quadratic programming but is less general than semidefinite programming, see Figure 4.3 for a better understanding.

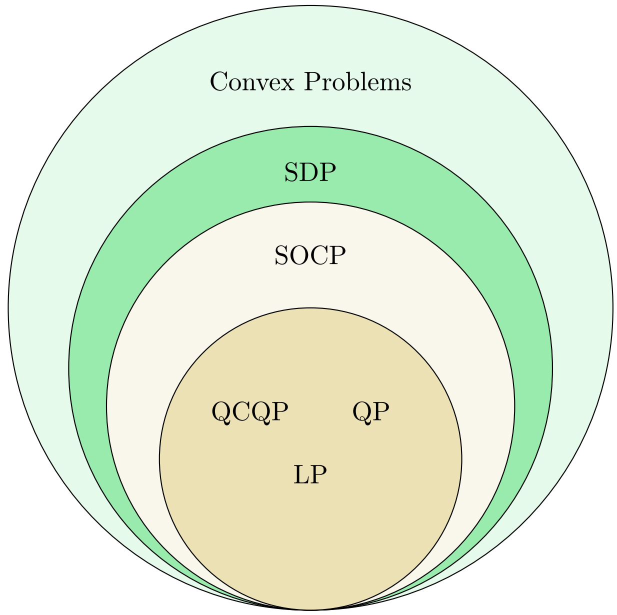

Figure 4.3: Hierarchy of main convex problems

Definition 4 *(SOCP): Second-order cone programming refers to optimization problems having the form*

$$\begin{aligned} &\textit{minimize} \quad f^T x \\ &\textit{subject to} \quad ||A_i x + b_i|| \leq c_i^T x + d_i, \quad i = 1, ..., m \end{aligned}$$

Where $x \in \mathbb{R}^n$ is the optimization variable, and all other quantities, $f \in \mathbb{R}^n, A_i \in \mathbb{R}^{k_i \times n}, b_i \in \mathbb{R}^{k_i}, c_i \in \mathbb{R}^n$, and $d_i \in \mathbb{R}$ are data. The norm is the Euclidean norm: $||u|| = (u^T u)^{1/2}$ and constraints of the form

$$||Ax + b|| \leq c^T x + d$$

are called second-order cone constraints because the affinely defined variables $u = Ax + b \in \mathbb{R}^k$ and $t = c^T x + d \in \mathbb{R}$ are constrained to belong to the second-order cone defined by

$$\{(u, t) \in \mathbb{R}^{k+1} : ||u|| \leq t\} \tag{4.1}$$

There are general families of problems that can be recasted as SOCPs. These include robust linear programming and robust least squares problems, problems involving sums or maxima of norms, or with convex hyperbolic constraints. A variety of engineering applications, such as filter design, antenna array weight design, truss design, and grasping forte optimization in robotics can be solved through SOCP. In reference [35] several problems for electric power system are modeled with SOCP. Optimal power flow problem is one of the problems that can be formulated with SOCP.

4.2 Economic dispatch

As aforementioned in Section (3.1), the main problem of the economic dispatch is the treatment of solar radiance and market-prices uncertainties. The incursion of these random variables in math model have been mainly approached from two different methods. On the one hand, methods that consider a stochastic programming approach [17][36][37][38], and on the other hand methods that use a robust optimization approach (RO) [39][40].

Since robust optimization can handle uncertainties covering up the worst cases and it can be expressed as an convex optimization problem the RO approach is used to model random variables. The model proposed here is different from the aforementioned approaches because it does not use neither different levels to include uncertainties as in stochastic programming which creates a large number of scenarios nor uncertainties sets as in some cases of robust optimization when probability distribution of random variable is hard to obtain.

4.2.1 Market-price uncertainty

The electric market prices are related in the objective function of the deteministic day-ahead model Definition (1) as C_m, in these formulation C_m is a known parameter but in real case the market prices are a random variable and its behavior along time is shown in Figure 4.4, from that is possible to assume a normal distribution to market price variable at each hour of day and obtain confidence bound of variable $C_{m(t)} \sim N(\mu_t, \sigma_t^2)$ that is used in the following formulation.

Model 3 *A robust representation of costs for the day-ahead-model*

$$\max_{\psi} \ \overline{C}_m^T P_{vpp} - \gamma(P_{vpp}^T \Sigma P_{vpp}) - \sum_{t=1}^{24} \left[C_{DSO\_d} \cdot dieselGencost_{(t)}\right]$$

Subject to

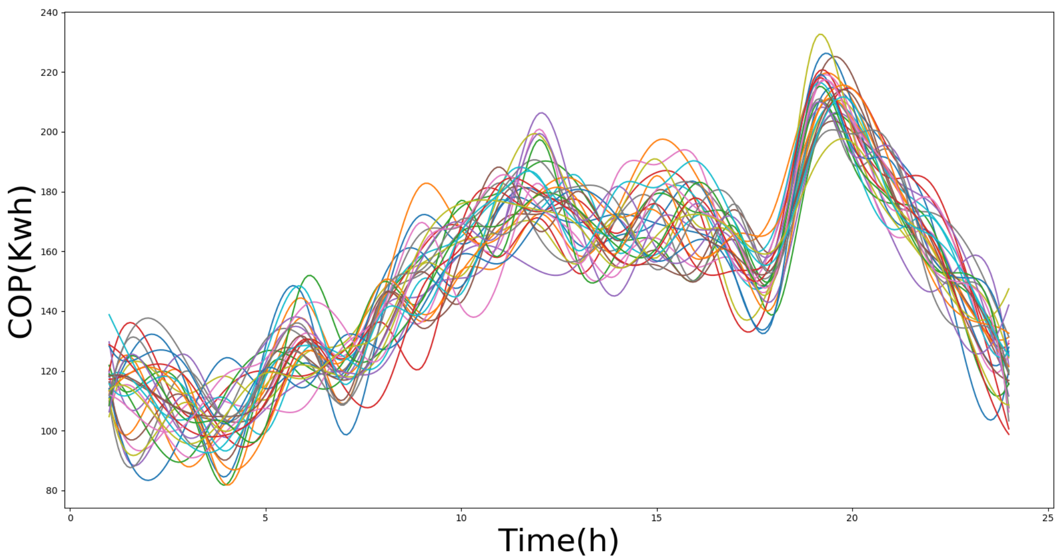

Figure 4.4: Market prices vs. time for September 2016. Data taken from [41]

$$\sum_{d=1}^{Nd} P_{d(t)} + \sum_{s=1}^{Ns} P_{s(t)} + \sum_{pv=1}^{Npv} P_{pv(t)} = P_{vpp(t)} \qquad P_d \in \mathcal{D}, P_s \in \mathcal{B}, P_{pv} \in \mathcal{PV}$$

$$P_{s(t)} = \frac{\Delta E_{s(t)}}{\Delta t} \qquad P_s \in \mathcal{B}, \Delta t = 1$$

$$|P_{s(t)}| \leq P_{s\_max} \qquad P_s \in \mathcal{B}$$

$$\Delta E_{s(t)} = E_{s(t-1)} - E_{s(t)}$$

$$E_{s(1)} = E_{s(24)} = E_{fixed}$$

$$0 \leq E_{s(t)} \leq E_{s\_max}$$

$$0 \leq P_{d(t)} \leq P_{d\_max} \quad P_d \in \mathcal{D}$$

Where new variables $\overline{C}_m$ is the mean of C_m, new parameter $\gamma > 0$ is added and it is called the *risk aversion parameter* and Σ is the covariance of C_m.

The first two terms of the objective function can be interpreted as the maximization of the expected value of power sold to the market and the penalization to its variance respectively.

To achieve Model 3 it is considered the linear program of deterministic day ahead Model 1 to include uncertainties with $P_{vpp} \in \mathbf{R}^{24}$ and the *random* vector cost $C_m \in \mathbf{R}^{24}$, its mean value $\overline{C}_m$ and covariance $\mathbf{E}(C_m - \overline{C}_m)(C_m - \overline{C}_m)^T = \Sigma$.

For a given $P_{vpp} \in \mathbf{R}^{24}$ the cost $C_m^T \cdot P_{vpp}$ is a random variable with mean

$$\mathbf{E}(C_m^T \cdot P_{vpp}) = \overline{C}_m^T \cdot P_{vpp}$$

and variance

$$\mathbf{var}(C_m^T \cdot P_{vpp}) = \mathbf{E}(C_m^T \cdot P_{vpp} - \mathbf{E}C_m^T \cdot P_{vpp})^2 = P_{vpp}^T \Sigma P_{vpp}$$

In general there is a trade-off between small expected cost and small cost variance. One way to take variance into account is to minimize a linear combination of the expected value and the variance of the cost as follow:

$$\mathbf{E}(C_m^T \cdot P_{vpp}) + \gamma \mathbf{var}(C_m^T \cdot P_{vpp})$$

Which is called *risk-sensitive cost* and $\gamma > 0$ parameter is called **risk aversion parameter**. To minimize the risk-sensitive cost of VPP power the following quadratic convex problem is formulated:

$$\max_{\psi} \ \overline{C}_m^T P_{vpp} - \gamma(P_{vpp}^T \Sigma P_{vpp}) - \sum_{t=1}^{24} \left[C_{DSO\_d} \cdot dieselGencost_{(t)} \right]$$

Subject to

$$\sum_{d=1}^{Nd} P_{d(t)} + \sum_{s=1}^{Ns} P_{s(t)} + \sum_{pv=1}^{Npv} P_{pv(t)} = P_{vpp(t)} \qquad P_d \in \mathcal{D}, P_s \in \mathcal{B}, P_{pv} \in \mathcal{PV}$$

$$P_{s(t)} = \frac{\Delta E_{s(t)}}{\Delta t} \qquad P_s \in \mathcal{B}, \Delta t = 1$$

$$|P_{s(t)}| \leq P_{s\_max} \qquad P_s \in \mathcal{B}$$

$$\Delta E_{s(t)} = E_{s(t-1)} - E_{s(t)}$$

$$E_{s(1)} = E_{s(24)} = E_{fixed}$$

$$0 \leq E_{s(t)} \leq E_{s\_max}$$

$$0 \leq P_{d(t)} \leq P_{d\_max} \quad P_d \in \mathcal{D}$$

In this way the uncertainties in costs are included in the model through an objective function that maximize the expected value of cost penalizing its variance with the risk aversion parameter.

γ is used to choose the risk that the VPP assumes offering power in energy market, where the prices $C_{m(t)} \sim N(\mu_t, \sigma_t^2)$.

4.2.2 Solar radiance uncertainty

The solar radiance is a random variable that changes depending the hour of the day and the month of the year. In Figure 4.5 the solar radiance behavior along day during the month of september in 2016 is shown. These real data were taken from hidro-climatological network of Risaralda[42]. Also in Figure 4.6 the histogram for radiation along that is presented. From that it is possible to assume a normal distribution to solar radiance for each hour of the day.

The uncertainties in solar radiance affects directly the photo-voltaic generation variable P_{pv} so at each hour of day it is assumed that $P_{pv(t)} \sim N(\mu_t, \sigma_t^2)$.

The deterministic day-ahead Model 1 uses $P_{pv(t)}$ in the following power balance constraint:

$$\sum_{d=1}^{Nd} P_{d(t)} + \sum_{s=1}^{Ns} P_{s(t)} + \sum_{pv=1}^{Npv} P_{pv(t)} = P_{vpp(t)} \qquad P_d \in \mathcal{D}, P_s \in \mathcal{B}, P_{pv} \in \mathcal{PV} \tag{4.2}$$

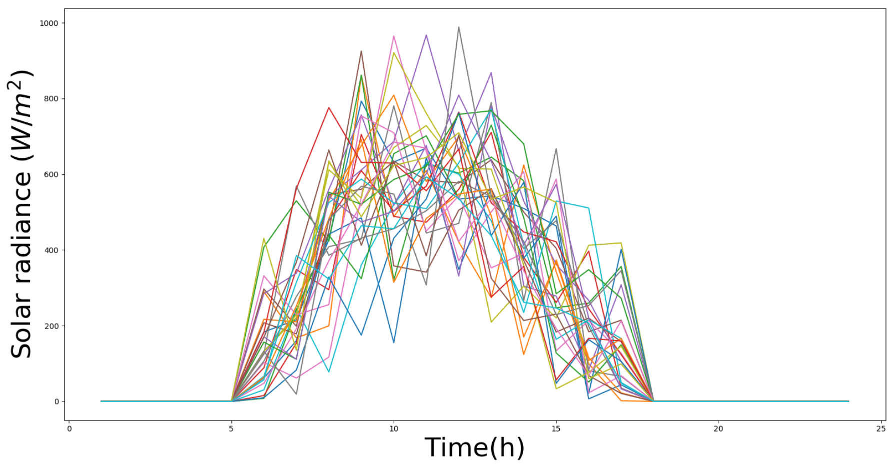

Figure 4.5: Solar radiance in September 2016. Data taken from [42]

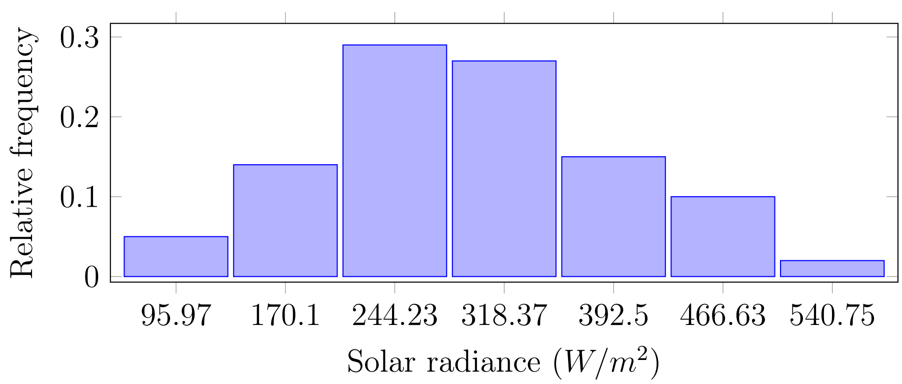

Figure 4.6: Solar radiance at 9 am in september 2016. Data taken from [42]

To include the random behavior of photo-voltaic generation it is proposed the following change in the power balance Constraint 4.2, rewriting it as the follow linear constraint:

$$P_{vpp(t)} - \sum_{s=1}^{Ns} P_{s(t)} - \sum_{d=1}^{Nd} P_{d(t)} < \phi^{-1}(1-n) \tag{4.3}$$

Where $\phi^{-1}(1-n)$ is the *quantile function* or *percent-point function* of the normal distribution related to $P_{pv(t)}$ and parameter $0 \leq n \leq 1$ correspond to the probability with which it is expected that the constraint will be fulfilled exceeding n.

Starting from the power balance constraint 4.2, That equality is relaxed to an inequality constraint and random term $P_{pv(t)}$ is left to the right

$$P_{vpp(t)} - \sum_{d=1}^{Nd} P_{d(t)} - \sum_{s=1}^{Ns} P_{s(t)} < \sum_{pv=1}^{Npv} P_{pv(t)}$$

with x as a vector of decisions variables, power balance constraint can be expresed as $g(x) < P_{pv}$ and it is defined the following probabilistic restriction:

$$Prob\,(g(x) < P_{pv}) \geq n \tag{4.4}$$

Where it defines the probability n with it is expected that the constraint will be fulfilled. Rewriting the random variable P_{pv} as X and the value $g(x)$ as X_0 and using a little of probability theory, it is perform the following:

$$\begin{aligned} Prob(X_0 < X) &\geq n \\ Prob(X > X_0) &\geq n \\ 1 - Prob(X \leq X_0) &\geq n \\ Prob(X \leq X_0) &\leq 1 - n \end{aligned}$$

Since the random variable $X \sim N(\mu, \sigma^2)$ using the definition of *quantile function*:

$$\phi^{-1}(n) = inf\{X_0 \in \mathbb{R}, n \leq Prob(X \leq X_0)\}$$

Where given a probability n it returns the value of X_0 satisfying $Prob(X \leq X_0) = n$, It is possible to obtain X_0 as follow:

$$X_0 = \phi^{-1}(1 - n)$$

Replacing in the power balance constraint the equivalence of random variable it is obtained that:

$$P_{vpp(t)} - \sum_{s=1}^{Ns} P_{s(t)} - \sum_{d=1}^{Nd} P_{d(t)} < \phi^{-1}(1 - n)$$

With that constraint it is included the random behavior of photovoltaic generation in the VPP day-ahead model. Parameter n is used to set the minimum probability with which the constraint is expected to be fulfilled.

4.2.3 Model with stochastic robust approximation

Finally the economic dispatch model with robust approximation of uncertainties in solar radiance and market prices are defined as follow:

Model 4 *Stochastic Robust approximation math model of economic dispatch*

$$\max_{\psi} \; \overline{C}_m^T P_{vpp} - \gamma(P_{vpp}^T \Sigma P_{vpp}) - \sum_{t=1}^{24} \left[C_{DSO\_d} \cdot dieselGencost_{(t)} \right] \tag{4.5}$$

subject to

$$P_{vpp(t)} - \sum_{s=1}^{Ns} P_{s(t)} - \sum_{d=1}^{Nd} P_{d(t)} < \phi^{-1}(1-n), \quad P_d \in \mathcal{D}, P_s \in \mathcal{B} \tag{4.6}$$

$$P_{s(t)} = \frac{\Delta E_{s(t)}}{\Delta t}, \quad P_s \in \mathcal{B} \tag{4.7}$$

$$|P_{s(t)}| \leq Ps_{max}| \tag{4.8}$$

$$\Delta E_{s(t)} = E_{s(t-1)} - E_{s(t)} \tag{4.9}$$

$$E_{s(1)} = E_{s(24)} = E_{fixed} \tag{4.10}$$

$$0 \leq E_{s(t)} \leq Es_{max} \tag{4.11}$$

$$0 \leq P_{d(t)} \leq Pd_{max}, \quad P_d \in \mathcal{D} \tag{4.12}$$

This model has a quadratic function that is convex and a set of linear and affine constraint that form a convex space, so this is a convex optimization problem that guaranty unique solution.

4.3 Convex relaxation of the power flow equations

According to [33] non-linearity and non-convexity of the optimal operation Model 2 comes from one of the following three forms: 1) $e_i^2 + f_i^2$, 2) $e_i e_j + f_i f_j$, 3) $e_i f_j - f_i e_j$, new variables are include to catch this non-linearity. For each bus i and each distribution line (i, j) the new variables C_{ii}, C_{ij}, S_{ij} are defined as follow:

- $C_{ii} = e_i^2 + f_i^2$
- $C_{ij} = e_i e_j + f_i f_j$
- $S_{ij} = e_i f_j - e_j f_i$

These new variables satisfy the relation $C_{ij}^2 + S_{ij}^2 = C_{ii}C_{jj}$ and with a change of variables, it is defined the exact alternative formulation of the optimal operation problem as follows:

$$\text{Minimize} \quad \sum_{i \in G} C_i(P_i^g) \tag{4.13}$$

Subject to

$$P_i^g - P_i^d = G_{ii}C_{ii} + \sum_{j \in \delta(i)} (G_{ij}C_{ij} - B_{ij}S_{ij}) \quad i \in \mathcal{N} \tag{4.14}$$

$$Q_i^g - Q_i^d = -B_{ii}C_{ii} + \sum_{j \in \delta(i)} [-B_{ij}C_{ij} - G_{ij}S_{ij}] \quad i \in \mathcal{N} \tag{4.15}$$

$$\underline{V}_i^2 \leq C_{ii} \leq \overline{V}_i^2 \quad i \in \mathcal{N} \tag{4.16}$$

$$C_{ij} = C_{ji}, \quad S_{ij} = -S_{ji} \quad (i,j) \in \mathcal{L} \tag{4.17}$$

$$C_{ij}^2 + S_{ij}^2 = C_{ii}C_{jj} \quad (i,j) \in \mathcal{L} \tag{4.18}$$

$$P_i^{min} \leq P_i^g \leq P_i^{max} \quad i \in \mathcal{K} \tag{4.19}$$

$$Q_i^{min} \leq Q_i^g \leq Q_i^m ax \quad i \in \mathcal{K} \tag{4.20}$$

This exact formulation of the optimal operation was introduced in [43] and [44]. Here is important to note that this formulation is still non-convex by constraint 4.18 which can be relaxed as follow:

$$C_{ij}^2 + S_{ij}^2 \leq C_{ii}C_{jj} \qquad (i,j) \in \mathcal{L} \tag{4.21}$$

The non-linear right term can be expressed as:

$$C_{ii}C_{jj} = \left(\frac{C_{ii} + C_{jj}}{2}\right)^2 - \left(\frac{C_{ii} - C_{jj}}{2}\right)^2$$

And replacing the equivalence of $C_{ii}C_{jj}$ in equation 4.21 the non-linear term is dropped as follow:

$$C_{ij}^2 + S_{ij}^2 + \left(\frac{C_{ii} - C_{jj}}{2}\right)^2 \leq \left(\frac{C_{ii} + C_{jj}}{2}\right)^2 \qquad (i,j) \in \mathcal{L} \tag{4.22}$$

Which represents a rotated SOCP cone in $\mathbb{R}^4$ and therefore if it is defined u as:

$$u = C_{ij} + S_{ij} + \frac{(C_{ii} - C_{jj})}{2}$$

and t as:

$$t = \frac{C_{ii} + C_{ii}}{2}$$

the constraint 4.22 can be expressed as:

$$||u|| \leq t$$

Which correspond to the *second-order cone constraint* 4.1.

Finally relaxing the non-linear constraint 4.18 with the second-order cone constraint 4.22 the resulting model corresponds to *the second-order cone relaxation model of the optimal operation problem* valid for radial networks and used in the real time operation of the VPP since this is a convex optimization problem where the voltage phase angles θ_i's are recovered solving the following system of linear equation with the optimal solution of C_{ij}, S_{ij}:

$$\theta_j - \theta_i = atan2(S_{ij}, C_{ij}) \quad (i,j) \in \mathcal{L}$$

And the $atan2$ function is defined as follow:

$$atan2(y,x) = \begin{cases} arctan\frac{y}{x} & \text{if } x \geq 0 \\ arctan\frac{y}{x} + \pi & \text{if } y \geq 0, x < 0 \\ arctan\frac{y}{x} - \pi & \text{if } y < 0, x < 0 \\ +\frac{\pi}{2} & \text{if } y > 0, x = 0 \\ -\frac{\pi}{2} & \text{if } y < 0, x = 0 \\ \text{undefined} & \text{if } y = 0, x = 0 \end{cases}$$

The accuracy of the second-order cone relaxation for optimal operation problem can be seen in [33] where different implementations are compared.

4.4 Receding horizon control (RHC)

The control of the VPP operation in real time is based on the RHC idea also known as model predictive control which is a general purpose control scheme that involves to solve repeatedly a constrained optimization problem, using predictions of some variables (i.e. future electricity costs and solar radiance) and constraints over a moving time horizon to choose the control action (Figure 4.7 shows the concept).

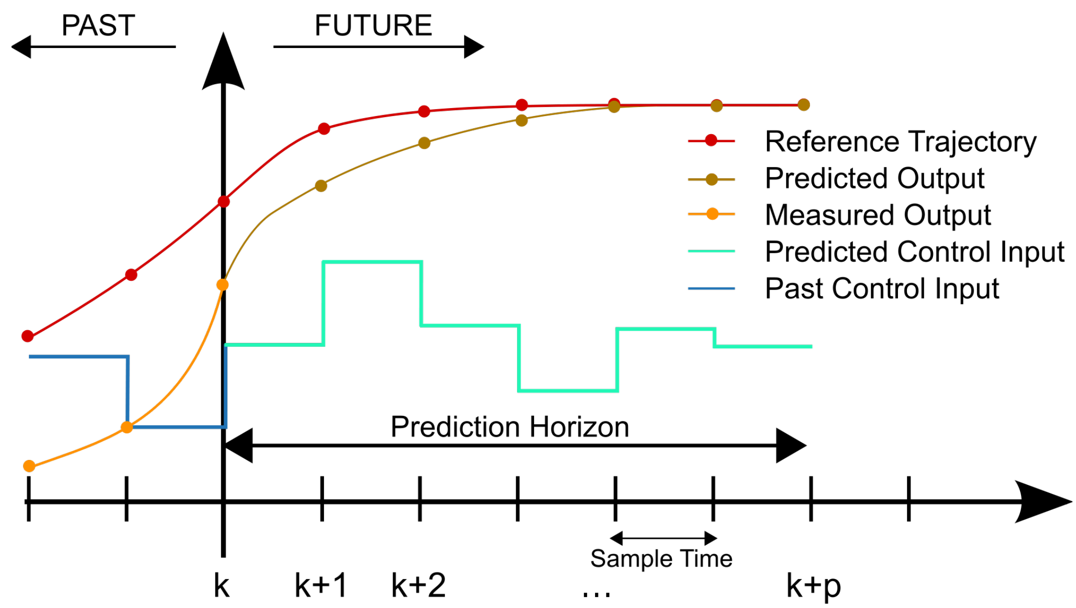

Figure 4.7: Concept of receding horizon control

In the context of the VPP operation the reference is the scheduled power profile previously obtained in the economic dispatch problem, the sample time is one hour, the prediction horizon can be 4 hours and the state of the VPP in real time is the signal to control. In that sense the optimization problem of real time operation is executed at time i over a fixed future interval $[i,\ N-1\ ]$ taking into account the current and future constraints then the first step in the resulting optimal control is applied (which correspond to time i) and the process is repeated with the fixed horizon at time $i+1$ in the interval $[i+1,\ N\ ]$ and so on.

4.5 Real time operation model

The math model for real time operation involves the solution of the power flow equations and constraints about DERs overs a fixed time window (tw) that correspond to the receding horizon control method. This model is executed each hour along the day with the aim of to prevent deviation from scheduled power while technical and physical constraints are satisfied.

The parameters and variables of real time operation model can be found in Table 4.1 and Table 4.2 respectively:

tw $\in \mathbb{R}$ time window	$P_{diesel\_max} \in \mathbb{R}^N$ max diesel P power
N $\in \mathbb{R}$ nodes in system	$S_{diesel\_max} \in \mathbb{R}^N$ max diesel S power
$1_{tw} \in \mathbb{R}^{tw}$ vector of ones	$P_{st\_max} \in \mathbb{R}^N$ max batt P power
$1_N \in \mathbb{R}^N$ vector of ones	$P_{slack\_min} \in \mathbb{R}^N$ min slack P power
$E_{init} \in \mathbb{R}^N$ batt energy at init of frame	$S_{st\_max} \in \mathbb{R}^N$ max batt S power
$E_{end} \in \mathbb{R}^N$ batt energy at end of frame	$G \in \mathbb{R}^{N\times N}$ G matrix
$P_{pv\_max} \in \mathbb{R}^{tw\times N}$ max photo-voltaic P	$B \in \mathbb{R}^{N\times N}$ B matrix
$S_{pv\_max} \in \mathbb{R}^{tw\times N}$ max photo-voltaic S	$P_d \in \mathbb{R}^{tw\times N}$ demand P power
$P_{slack\_max} \in \mathbb{R}^N$ max slack P power	$Q_d \in \mathbb{R}^{tw\times N}$ demand Q power
$Q_{slack\_min} \in \mathbb{R}^N$ min slack Q power	$V_{min} \in \mathbb{R}$ technical min Voltage
$Q_{slack\_max} \in \mathbb{R}^N$ max slack Q power	$V_{max} \in \mathbb{R}$ technical max Voltage
$P_{mg\_max} \in \mathbb{R}^{tw\times N}$ max P of Microgrid	α_d,$\beta_d \in \mathbb{R}$ diesel Gen parameters
$P_{mg\_min} \in \mathbb{R}^{tw\times N}$ min Microgrid P	$C_{DSO\_d} \in \mathbb{R}$ DSO diesel gen Cost
$S_{mg\_max} \in \mathbb{R}^{tw\times N}$ max S of microgrid	$C_{lt\_d} \in \mathbb{R}$ diesel lt cost
$\rho \in \mathbb{R}$ penalization factor	$C_{slack} \in \mathbb{R}$ DSO energy Cost
$P_{vpp\_da} \in \mathbb{R}^{tw}$ VPP day-ahead	$C_m \in \mathbb{R}^{tw}$ market-prices
$1_{tw\_N} \in \mathbb{R}^{tw\times N}$ ones matriz	$C_{DSO} \in \mathbb{R}$ DSO energy Cost

Table 4.1: Parameters of real time operation model

Finally the math model for real time operation of VPP, that include technical and physical constraints of the grid and prevent deviation from scheduled power is as follow:

Model 5 *VPP real time operation model*

$$minimize \quad \rho \cdot ||P_{vpp} - P_{vpp\_da}|| + Cost\_slack + Cost\_diesel \tag{4.23}$$

Subject to:

$$P_{vpp} = P_{diesel} \cdot 1_N + P_{pv} \cdot 1_N + P_{st} \cdot 1_N + P_{mg} \cdot 1_M \tag{4.24}$$

$P_{vpp} \in \mathbb{R}^{tw}$, vpp power	$\mathrm{S} \in \mathbb{R}^{tw\times N\times N}$, soc relaxation vars
$P_{diesel} \in \mathbb{R}^{tw\times N}$, diesel active power	$\mathrm{C} \in \mathbb{R}^{tw\times N\times N}$, soc relaxation vars
$Q_{diesel} \in \mathbb{R}^{tw\times N}$, diesel reactive power	$\mathrm{Q} \in \mathbb{R}^{tw\times N}$, nodes reactive power
$P_{pv} \in \mathbb{R}^{tw\times N}$, photo-voltaic active power	$\mathrm{P} \in \mathbb{R}^{tw\times N}$, nodes active power
$Q_{pv} \in \mathbb{R}^{tw\times N}$, photo-voltaic reactive power	$E_{st} \in \mathbb{R}^{tw\times N}$, batteries energy
$P_{slack} \in \mathbb{R}^{tw\times N}$, slack active power	$P_{st} \in \mathbb{R}^{tw\times N}$, batteries active power
$Q_{slack} \in \mathbb{R}^{tw\times N}$, slack reactive power	$Q_{st} \in \mathbb{R}^{tw\times N}$, batteries reactive power
$P_{mg} \in \mathbb{R}^{tw\times N}$, microgrid P power	$Q_{mg} \in \mathbb{R}^{tw\times N}$, microgrid Q power

Table 4.2: Variables of real time operation model

$$P = P_{slack} + P_{diesel} + P_{pv} + P_{st} + P_{mg} \tag{4.25}$$

$$Q = Q_{slack} + Q_{diesel} + Q_{pv} + Q_{st} + Q_{mg} \tag{4.26}$$

$$E_{st}(0) = E_{init}, \quad E_{st}(tw) = E_{end}, \quad E_{st} \geq 0 \tag{4.27}$$

$$P_{st} = A \cdot E_{st}, \quad |P_{st}| \leq 1_{tw} P_{st\_max}^T \tag{4.28}$$

$$1_{tw} \cdot P_{slack\_min}^T \leq P_{slack} \leq 1_{tw} \cdot P_{slack\_max}^T \tag{4.29}$$

$$1_{tw} \cdot Q_{slack\_min}^T \leq Q_{slack} \leq 1_{tw} \cdot Q_{slack\_max}^T \tag{4.30}$$

$$0 \leq P_{diesel} \leq 1_{tw} \cdot P_{diesel\_max}^T \tag{4.31}$$

$$0 \leq P_{pv} \leq P_{pv\_max} \tag{4.32}$$

$$0 \leq P_{mg} \leq P_{mg\_max} \tag{4.33}$$

$$P_{diesel}^2 + Q_{diesel}^2 \leq 1_{tw} \cdot {S_{diesel\_max}^2}^T \tag{4.34}$$

$$P_{pv}^2 + Q_{pv}^2 \leq S_{pv\_max}^2 \tag{4.35}$$

$$P_{st}^2 + Q_{st}^2 \leq 1_{tw} \cdot {S_{st\_max}^2}^T \tag{4.36}$$

$$P_{mg}^2 + Q_{mg}^2 \leq 1_{tw} \cdot {S_{mg\_max}^2}^T \tag{4.37}$$

$$S(t) = -S(t)^T \quad t \in \{1, 2, ..., tw\} \tag{4.38}$$

$$P(t) = diag(G \cdot C(t)) - diag(B \cdot S(t)^T) + P_d(t) \quad t \in \{1, ..., tw\} \tag{4.39}$$

$$Q(t) = -diag(B \cdot C(t)) - diag(G \cdot S(t)^T) + Q_d(t) \quad t \in \{1, ..., tw\} \tag{4.40}$$

$$V_{min}^2 \cdot 1_N \leq diag(C(t)) \leq V_{max}^2 \cdot 1_N \quad t \in \{1, ..., tw\} \tag{4.41}$$

$$\|[C_{i,j}^t, S_{i,j}^t, (C_{i,i}^t - C_{j,j}^t)/2]\| \leq (C_{i,i}^t + C_{j,j}^t)/2 \quad t \in \{1, ..., tw\}, (i, j) \in \mathcal{L} \tag{4.42}$$

$$\|[C_{j,i}^t, S_{j,i}^t, (C_{j,j}^t - C_{i,i}^t)/2]\| \leq (C_{j,j}^t + C_{i,i}^t)/2 \quad t \in \{1, ..., tw\}, (i, j) \in \mathcal{L} \tag{4.43}$$

$$Cost\_slack = (C_{slack} \cdot 1_t w) \cdot (diag(1_{tw\_N} \cdot P_{slack}^T)) \tag{4.44}$$

$$\begin{aligned} Cost\_diesel = & C_{DSO\_d} \cdot (C_{lt\_d}[\alpha_d(Trace(1_{tw\_N} \cdot P_{diesel}^T))^2 \\ & + \beta_d(Trace(1_{tw\_N} * P_{diesel}^T))]) \end{aligned} \tag{4.45}$$

Where the objective Equation 4.23 is a quadratic function with ρ as a penalization parameter for deviation of scheduled power profile. Slack and diesel generation are included to minimize the cost of these generation and to prefer more cheap generation from other DERs. Equations (4.24) (4.25) (4.26) defines the VPP power, active and reactive power of system nodes respectively, Equations (4.27) (4.28) (4.36) are the batteries constraints. Similar in Equations (4.29) (4.30), (4.31) (4.34), (4.32) (4.35) and (4.33) (4.37) are defined the constraints for slack, diesel, photo-voltaic and microgrids generation respectively. Finally Equations (4.38) (4.39) (4.40) (4.41) (4.42) (4.43) are the second order cone relaxation constraint for optimal power flow problem in radial network.

It is important to say that parameters P_{mg_max}, S_{mg_max}, P_{mg_d} and Q_{mg_d} correspond to predicted power generation and power consumption by microgrids along time window in receding horizon control. That information are given by aggregator of microgrids. The way of how it is calculated exceeds the scope of this project but it is possible that the aggregator collect historical data and with artificial neural network obtain predicted values. By last it is important to see that the proposed VPP real time model (5) is still a convex model.

Chapter 5

Results

The proposed model of VPP for real time operation was tested in the IEEE69 power distribution test system that can be seen in Appendix (A.1) where the level of DER penetration over the grid is modeled through different scenarios and the behavior of the VPP is analyzed for distribute energy resources penetration of 10%, 40% and 70% respectively.

Before continuing with the study, it is important to note that in the economic dispatch model (4) the risk aversion parameter γ is crucial for the model and it is a parameter that the operator of the VPP must choose. Figure (5.1) shows the objective function for different risk aversion parameter values.

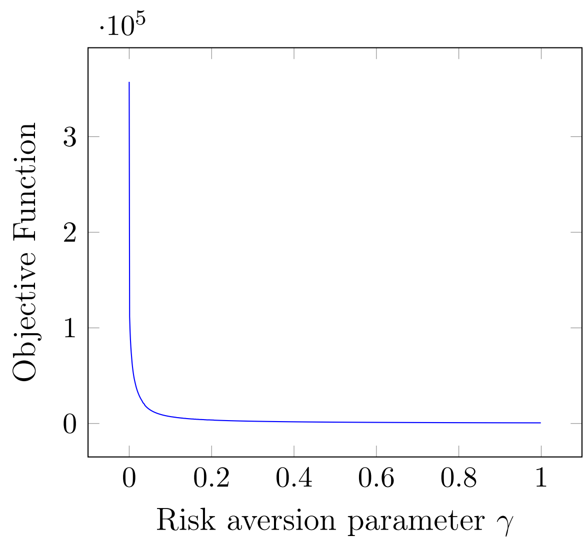

Figure 5.1: Impact of γ in the objective function

For γ close to 0.1 the model is very conservative preventing risk of deviation from expected value of $C_m P_{vpp}$ which is the value of power sold in energy market. On the other hand for γ values close to 0 the model does not take into account the risk of deviation, therefore a middle term for γ parameter is chosen setting $\gamma = 0.0001$ for the rest of the tests.

Another important parameter for the economic dispatch model is the probability $n > 0.5$ with it is expected that the probabilistic restriction of power balance (4.4) holds exceeding that value. Table (5.1) shows the parameters values used in simulations. All prices is shown in monetary units (m.u)

Description	Parameter	Value
Risk aversion	γ	0.0001
Probability restriction	n	0.8
Diesel cost generation	α_{d}	$1.38e-2$
Diesel cost generation	β_{d}	228.04
Diesel liter cost	$\mathrm{C_d}$	2271.87 m.u
VPP diesel gen cost	$\mathrm{C_{DSO_d}}$	1.5
DER energy cost	$\mathrm{C_{DSO}}$	50 m.u/kWh
Slack energy Cost	$\mathrm{C_{slack}}$	2000 m.u/kWh
Penalization to deviation	ρ	$1e6$

Table 5.1: parameters values

As the power production by diesel plants is more expensive than PV generation the VPP sets different cost for diesel generation (C_{DSO_d}) and photovoltaic or microgrid generation (C_{DSO}). Also, the energy costs in slack node (C_{slack}) is different and higher than renewable energy cost because the distribution system operator which is the owner of VPP is interested in sells energy while signs contracts with DER owners, on the other hand with a high cost in slack node it is prevented that the VPP model takes power from slack to prevent deviation from scheduled power.

If microgrid is connected to grid it can be seen as a generator or a load in a time window that consider the next 3 hours in the VPP control model, it is expected that this information comes from microgrid aggregator in real cases, so for simulations that information is generated randomly and since the proposed VPP does not manage demand the only variation in demands comes from microgrid penetration. Data is presented in per unit system since the math model use that system considering $1MVA$ as base.

5.1 Scenario 1

In this study case it is proposed a simple VPP that are composed by the distributed energy resources listed below in Table (5.2) and correspond to a penetration of 10% of DER over the grid shown in Figure (5.2).

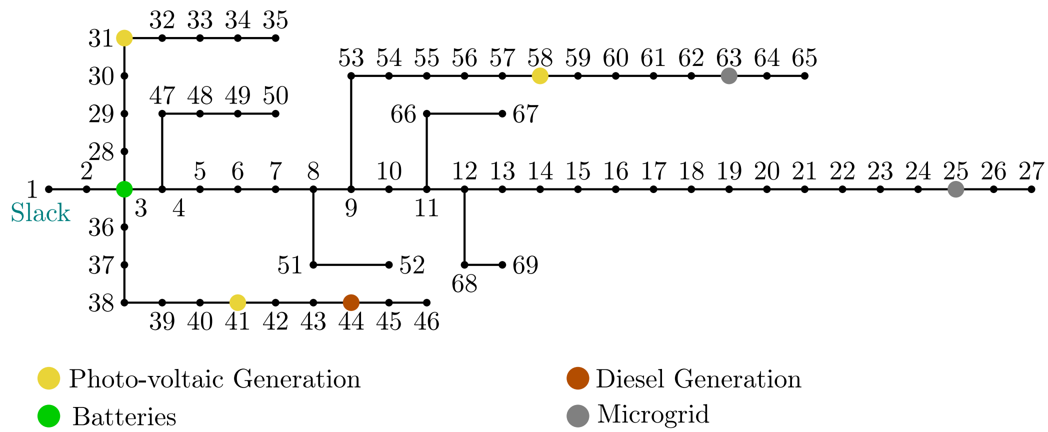

Figure 5.2: Test system for Scenario 1

Quantity	DER type	Installation Nodes	Max Unit Capacity
1	Diesel Power Plant	44	450 kW
1	Battery	3	50 kW
3	Photovoltaic Generation	31,41,58	114 kW
2	Microgrid	63,25	40 kW

Table 5.2: DER characteristics for scenario 1

The results for the economic dispatch of VPP are shown in Figure (5.3) where P_{vpp}, P_{st} and P_{pv} correspond to VPP, batteries and photovoltaic power. From that figure is easy to see the high dependency of VPP from photovoltaic generation since the maximum VPP power offered are related to the hours of the day where the solar radiance reaches the maximum values, also it can be evidenced that the VPP buys energy when the market prices are low (between 3am and 6am) and sells when the market prices are high (between 11am and 4pm), although the peak for market prices are at 6pm, 7pm and 8pm when photovoltaic generation is low the VPP uses the batteries for saving energy and sells it in those hours, that is the case for batteries power between 6pm and 9pm.

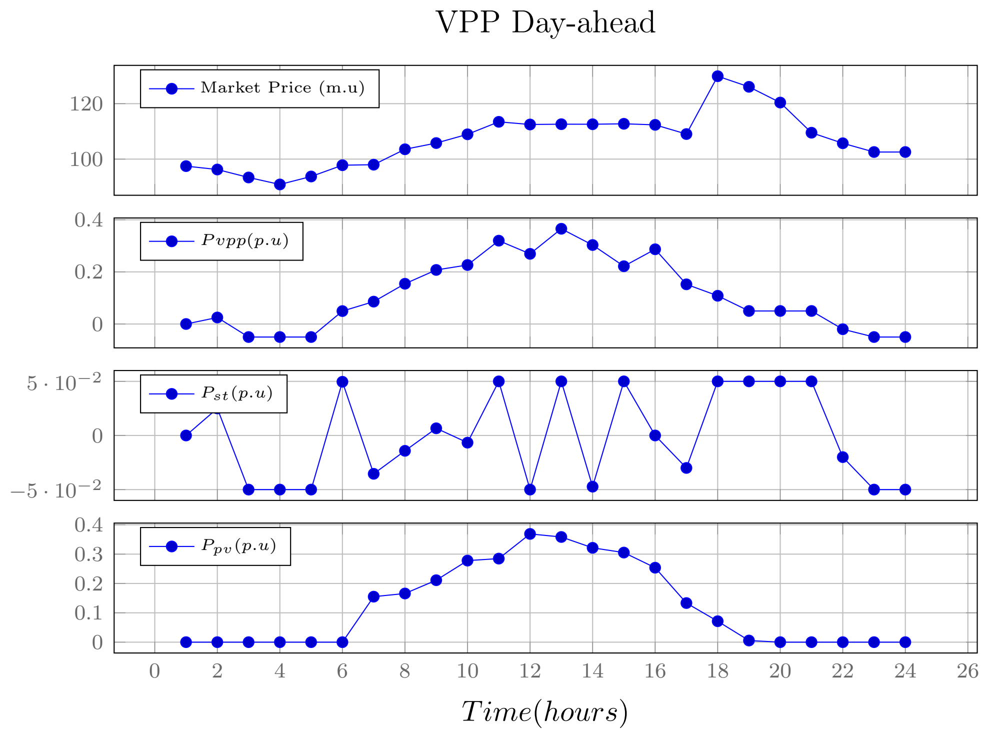

Figure 5.3: Day ahead for scenario 1

The figure for diesel power plant generation is not shown because it was not dispatched. The main information of the economic dispatch results are shown in Table (5.3) where it can be seen the profit of VPP given the expected value and variance of power sold and the power generation cost, finally the profit for VPP in economic dispatch is 158.643 m.u.

Description	Value (monetary units m.u)
Expected value of power sold	299.583
Variance in the power sold	8.149
Power generation cost	132.789
VPP Profit (objective function)	158.643

Table 5.3: Summary of economic dispatch for scenario 1.

The VPP real time operation in the next day is shown in Figure 5.4 where P_{vpp_da}, P_{vpp_rt} are the power of VPP in day-ahead and the power of VPP in real time, similarly P_{pv_da}, P_{pv_rt} are the photovoltaic generation in day-ahead and real time,P_{st}

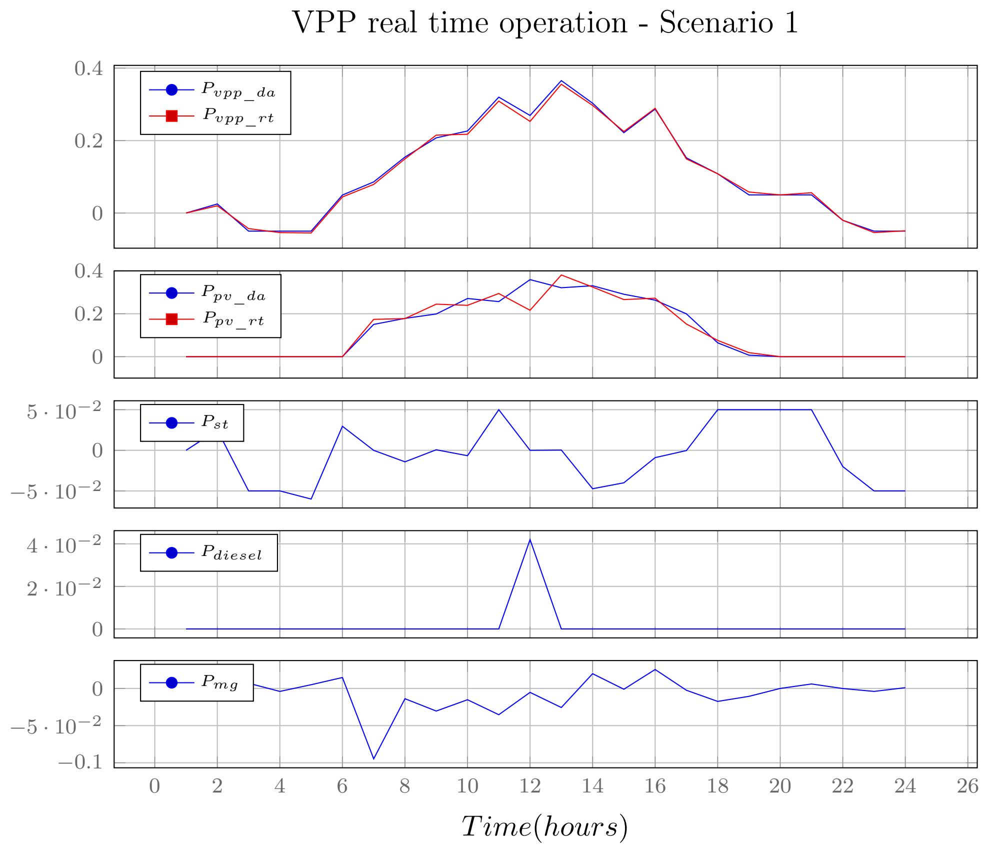

Figure 5.4: VPP operation in real time for scenario 1

and P_{mg} are the batteries and microgrid power. The error between the scheduled and real time operation is shown in Figure (5.5), from that figure the maximum deviation can be seen at 12m and 1pm that correspond to maximum deviation of photovoltaic generation, deviation from PV panels at that hour are compensated with power from diesel plants (P_{diesel}).

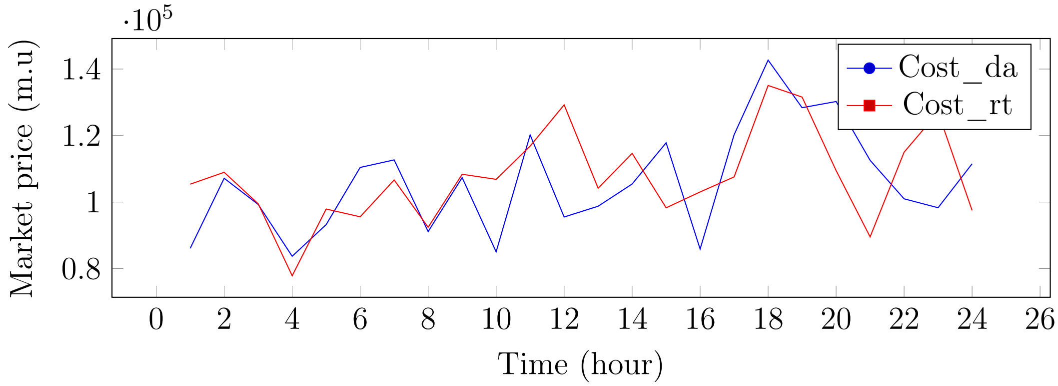

Figure 5.6: Variation of prices for scenario 1

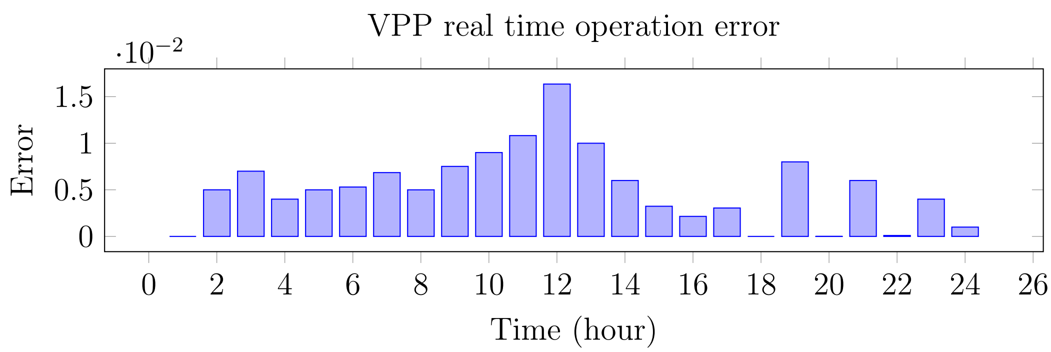

Figure 5.5: Error of VPP operation - scenario 1

The variation in prices is shown in Figure (5.6) and finally in Table (5.4) the power generation cost and profit of VPP are summarized where it is important to note that profit is less than predicted in the economic dispacth results because the high prices of diesel generation.

Description	Value
Cost of power sold	96.11%
Cost of power sold	124.93%
VPP profit	77.4%

Table 5.4: Generation costs and profit of VPP real time operation

5.2 Scenario 2

In this scenario it is considered a DER penetration of 40% over the grid. The resulting VPP is composed by 27 DERs summarized in Table 5.5. The DER distribution along the system can be seen in Figure (5.7).

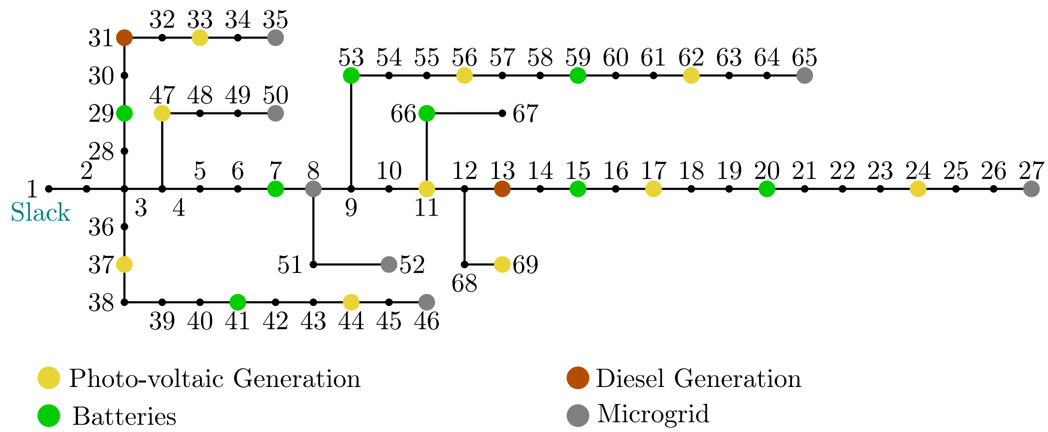

Figure 5.7: Test system for scenario 2

Quantity	DER type	Installation Nodes	Max Unit Capacity
2	Diesel Power Plant	31, 13	450 kW
8	Battery	29, 41, 53, 59, 15, 20, 66, 7	50 kW
10	Photovoltaic Generation	37, 47, 33, 44, 11, 56, 69, 17, 62, 24	114 kW
7	microgrid	35, 50, 8, 52, 76, 65, 27	40 kW

Table 5.5: DER distribution for scenario 2

The economic dispatch resulting information is shown in Figure (5.8), where the behavior of the VPP (P_{vpp}) is similar to Scenario 5.1 in the sense that it decides to buy energy when prices are low (at 4am) and to sell when the prices and DER power are optimal.

In the case of batteries power (P_{batt}) it saves the purchased energy at 4am when the prices are low and saves power when photovoltaic generation (P_{pv}) are high (i.e batteries power between 12m and 4pm). Finally the power offered between 6pm and

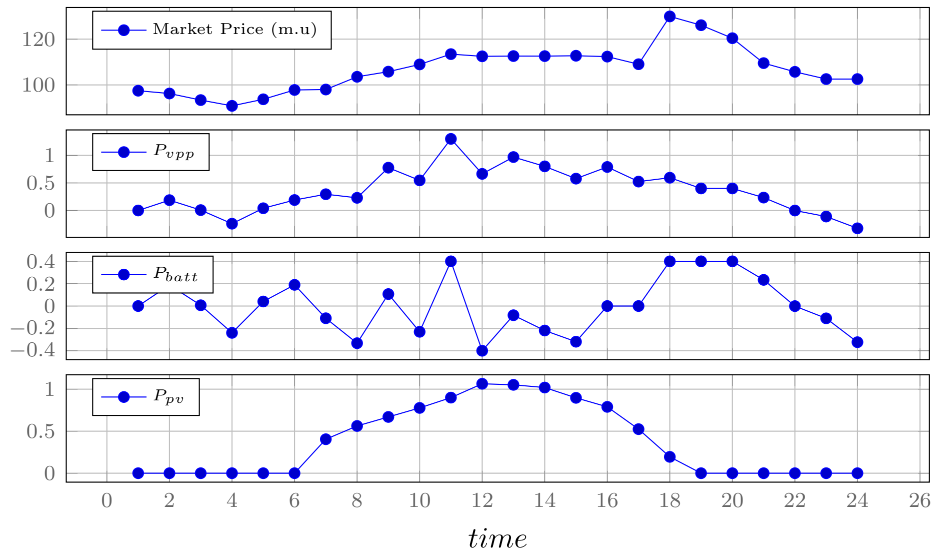

Figure 5.8: Day ahead for scenario 2

10pm is delivered by batteries since the power of PV panels is close to zero in these hours and the diesel power is not considered by its high generation cost.

Data about generation costs and profit of VPP in the economic dispath is shown in Table (5.6) where is expected a profit of 490.209 m.u.

Description	Value(m.u)
Expected value of power sold	1.003.051
Variance in the power sold	70.209
Power generation cost	442.632
VPP Profit (objective function)	490.209

Table 5.6: Summary of economic dispatch for scenario 2.

Figure (5.9) shows the related data with the real time operation given the day-ahead obtained previously, in this case the photovoltaic generation is less that predicted between 6am and 12m, and it is covered by the microgrid, batteries and diesel generation. The error between predicted and real power of VPP are shown in Figure (5.10)

The economical data related to the real time operation of VPP is shown in Table (5.7) where profit of VPP is 14.87% less than expected, even though the power sold in real time is 2.96% more than predicted. It is possible due to variation on energy prices in real time, Figure (5.11) shows the predicted and real time energy market prices where

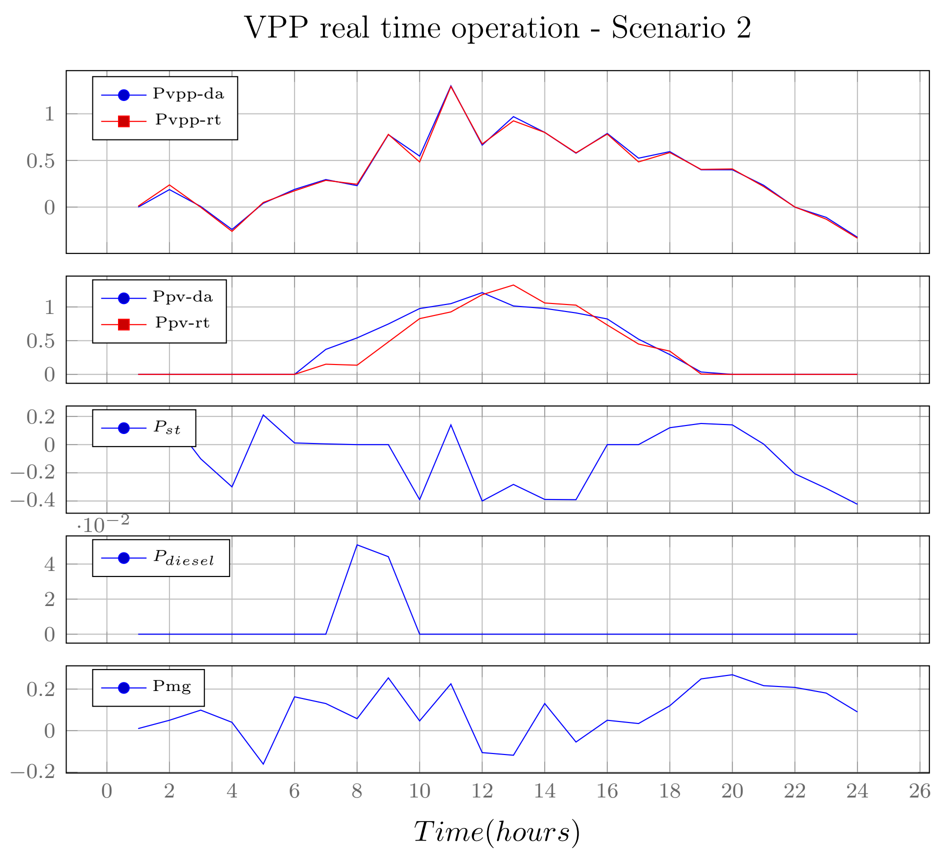

Figure 5.9: VPP operation in real time for scenario 2

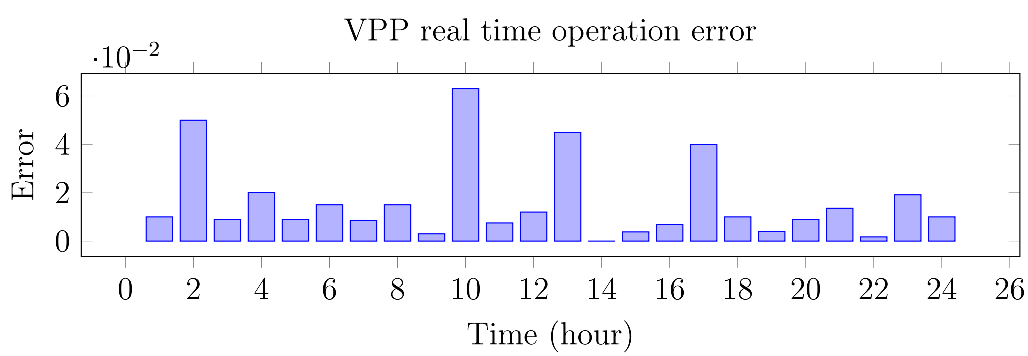

Figure 5.10: Error of VPP operation - scenario 2

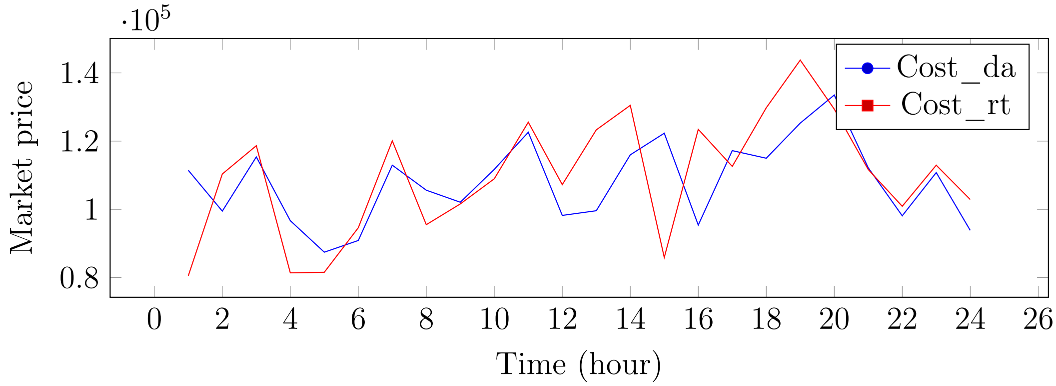

Figure 5.11: Variation of prices for scenario 2

the real time prices are greater than expected in different hours.

Description	Value
Cost of power sold	102.96%
Cost of generated power	139.05%
VPP profit	85.13%

Table 5.7: Generation costs and profit of VPP real time operation scenario 2

5.3 Scenario 3

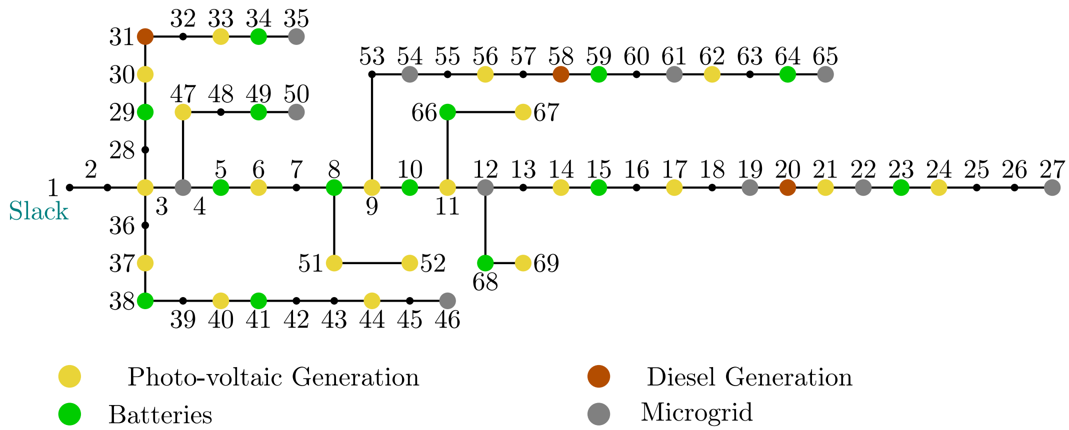

Figure 5.12: Scenario 3

In this scenario it is considered a DER penetration of 70% over the grid (see Figure 5.12), and the VPP is composed by 48 distributed energy resources listed in Table (5.8).

Quantity	DER type	Installation Nodes	Max Unit Capacity
3	Diesel Power Plant	31, 58, 20	450 kW
14	Battery	29, 38, 5, 34, 49, 41, 8, 10, 66, 68, 59, 15, 64, 23	50 kW
20	Photovoltaic Generation	30, 3, 37, 47, 33, 40, 6, 51, 9, 44, 52, 11, 56, 67, 69, 14, 17, 62, 21, 24	114 kW
11	Microgrid	3, 35, 50, 54, 46, 12, 61, 19, 65, 22, 27	40 kW

Table 5.8: Components of VPP for scenario 3

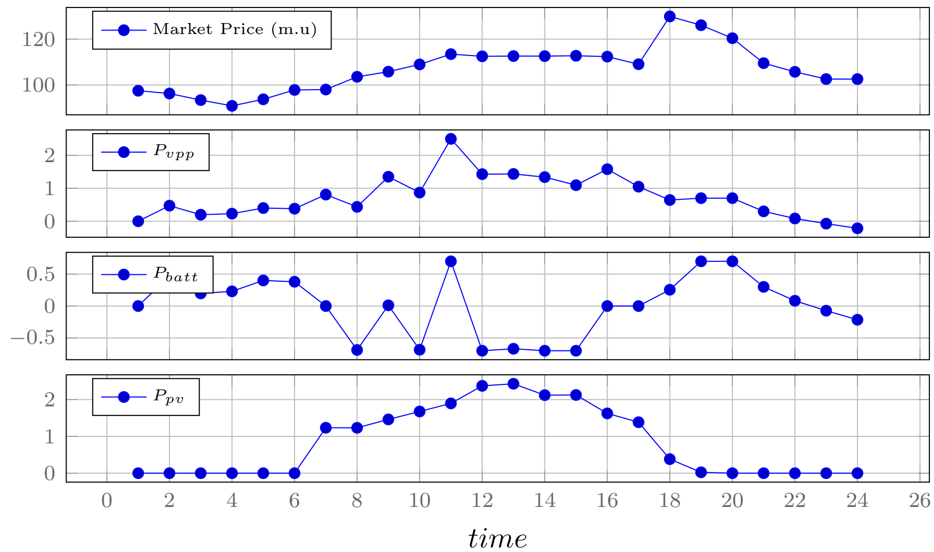

Figure 5.13: Day ahead for scenario 3

In Figure (5.13) it is shown the results of economic dispatch for this scenario, where the maximum power offered by VPP (P_{vpp}) is 2.457 p.u at 11am. The batteries system takes advantage of high energy market prices at 11am to deliver its power and it decides to save energy at hours 12m-3pm when photovoltaic generation reach the maximum power.

The costs and profit information is listed in Table (5.9) where it is expected to obtain a profit of 855.071 m.u given the predicted photovoltaic generation P_{pv}.

Description	Value (m.u)
Expected value of power sold	1.961.210
Variance in the power sold	220.874
Power generation cost	885.264
VPP Profit (objective function)	855.071

Table 5.9: Summary of economic dispatch for scenario 3.

The behavior of real time operation is shown in Figure (5.14), in this case the photovoltaic generation was below predicted in most hours therefore the diesel plants are dispatched at 7am,11am-3pm and 5pm also power from microgrids (P_{mg}) is used as much as possible to reduce deviation from scheduled values.

In this case is important to note that although the offered power in the economic dispatch of VPP is highly related to photovoltaic generation the scheduled profile is not the same that the PV panels profile, an example of that can be seen in Figure (5.14) where the photovoltaic generation at 2pm is 1.551 p.u and it is 0.165 p.u lower than predicted but it is 0.223 p.u more that the VPP scheduled power, the remaining power of PV panels are saved in the batteries and delivered after when PV generation is close to zero.

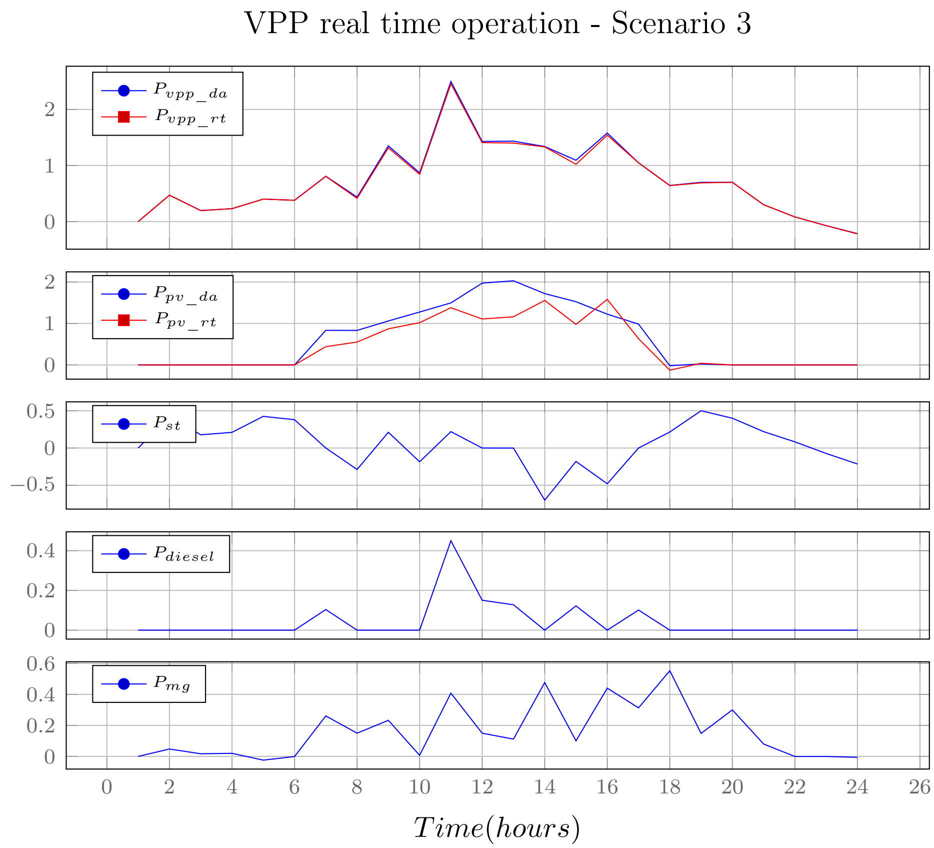

Figure 5.14: VPP operation in real time for scenario 3

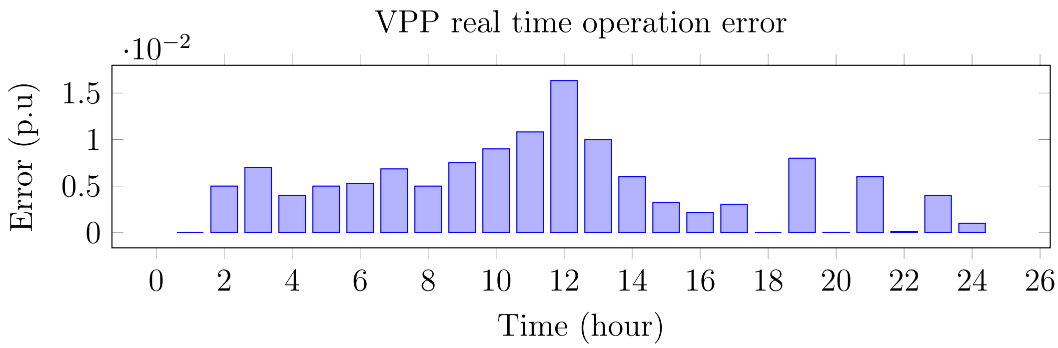

Figure 5.15: Error of VPP operation - scenario 1

The error between the offered power in economic distpach and the power in real time operation is shown in Figure (5.15) and data about the generation costs and profit of VPP are shown in Table (5.10) where the profit of VPP is 71.1% lower that prognosticated, similar the value of the power sold in market is 7.37% lower than predicted. That behavior comes from the difference between prognosticated and real energy prices shown in Figure (5.16) and the cost of diesel generation.

Description	Value
Cost of power sold	92.63%
Cost of generated power	177%
Profit of VPP	28.9%

Table 5.10: Generation costs and profit of VPP real time operation scenario 3

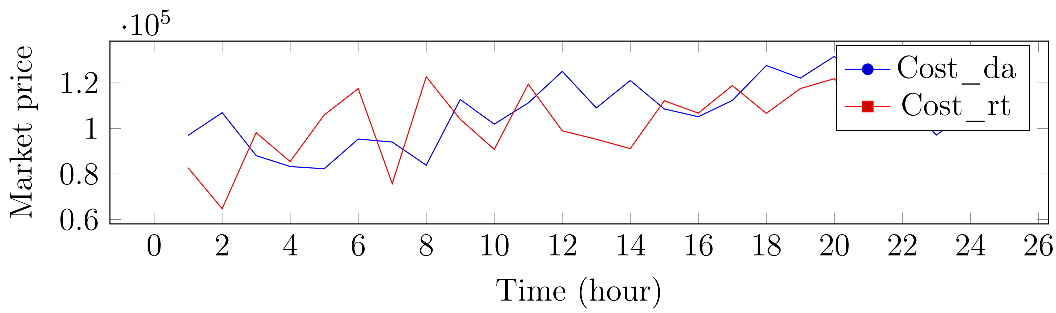

Figure 5.16: Variation of prices for scenario 3

5.4 Simulation framework

To perform the simulations it was chosen Python[45] as programming language with the numpy[46] library to support for large, multi-dimensional arrays and matrices operations and the cvxpy[47] library to model and solve convex optimization problems.

The computer specifications where simulations were made are listed in Table (5.11).

Component	Specification
Procesor	inter inside core i3
Memory	4GB
Operative system	Arch Linux (64 bits)

Table 5.11: Computer specifications

The execution time of different scenarios simulations are listed in Table (5.12) where is shown the mean execution time for solving the economic dispacth and the real time operation problems. With these execution times is valid to perform the operation and control of the VPP since the real time model is launched every hour and the economic dispatch is executed each 24 hours, On the other hand there is no significant difference from time execution point of view between the amount of DERs in VPP since the execution time depends on the size of the system and the size of the time window in control through receding horizon which was fixed to 4 hours.

Scenario	Economic Dispatch	real time Operation
1	0.114 s	3.239 s
2	0.112 s	3.265 s
3	0.115 s	3.221 s

Table 5.12: Average execution time of VPP models.

The VPP software architecture is shown in Figure 5.17 where a modular approach was used in its design, all components are written in Python which is a multi-platform language that can be executed in different operative systems over conventional computers or in single-board computers inclusive.

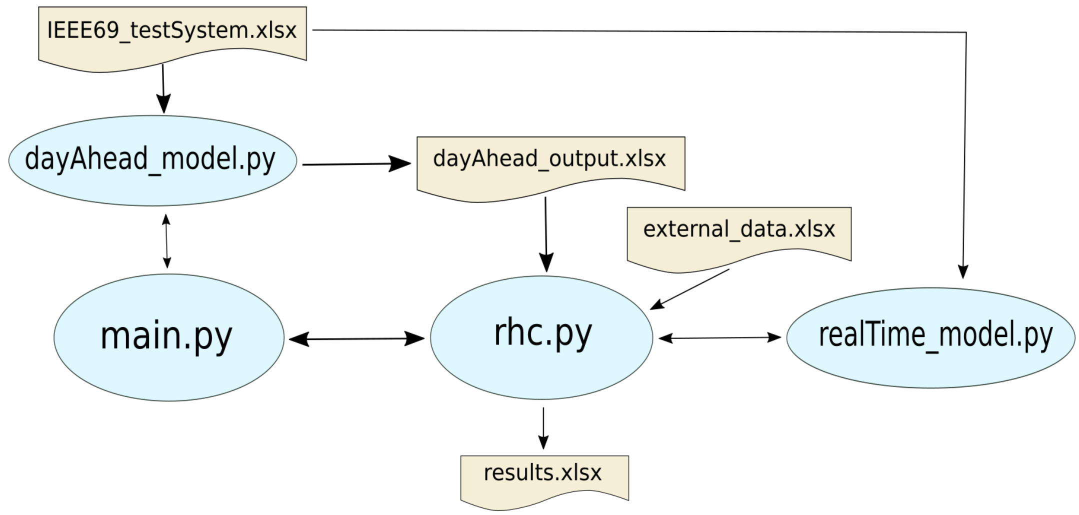

Figure 5.17: Architecture of VPP control software

The program is executed through *main.py* module which is the principal module that coordinates the others components. It calls every day the *dayAhead_ model.py* module that solves the economic dispatch model given the file *IEEE69_ testSystem.xlsx* which contains the info about the grid, the results are written in *dayAhead_ output.xlsx*. Also, the *main.py* component calls every hour the module *rhc.py* which perform the receding horizon control given the scheduled power, external data and grid information. It calls *realTime_ model.py* module that solve the real time operation model and return the results, then *rhc.py* writes in file *results.xlsx* the appropriate data.

Chapter 6

Conclusions

A model for real time operation of virtual power plants was proposed to support the high penetration of distributed energy resources along the grid. All distributed energy resources working coordinately in the economical dispatch problem can increase the profit of the VPP for day-ahead operation and in this way DER owners have visibility in the market through the VPP. Uncertainties in solar radiance and energy market prices were included in the final day-ahead convex model where the VPP through γ parameter can decide the level of risk that want to assume at the moment of bidding power in the electricity market.

In the real time operation of the VPP the scheduled power profile was followed throughout the day while technical and physical constraints were fulfilled; also the VPP mitigated deviations in photovoltaic generation with other resources like batteries or diesel plants, choosing the cheapest option. With ρ parameter the VPP model decided the penalization for deviation of scheduled power.

The computational burden of the VPP control and operation along the different DER penetration scenarios was approximately the same because with the matrix representation of the problem the execution time does not depend on the amount of DER, it depends directly on the size of grid and the size of time window taken in receding horizon control. That characteristic makes possible to support the high penetration of DER over the grid without increasing the execution time of VPP operation.

Finally, the use of convex optimization to model the economic dispatch and the real time operation problems has the advantages of reaching unique and optimal solution. In the case of real time operation the use of second order cone programming to include the non-convex and non-linear power flow equations gives the advantages of represent a NP-hard problem as a convex problem that gets solution in reasonable execution times (3 o 4 seconds) which in the power system context are considered real time. Also the use of sophisticated computers and devices as graphical processing units is not a requierement since the models run fast in conventional computers. On the other hand the resulting models of VPP can be expanded easily to support other technologies like electric vehicles and natural gas generators among others.

6.1 Future works and recomendations

An important topic in virtual power plants is the incursion of strategies for demand response and support the management of controllable loads which offer important technical service to different stakeholders.

Others control architectures for VPP can be explored to perform the operation in a distributed fashion without the problems of central controller. On the other hand the implementation of others convex relaxation for optimal power flow problem can be studied with the aim of use graphical processing units to reduce the execution time.

Finally other important future work is the study and simulation of VPP in the context of transmission system, considering its effect in terms of stability and market operation.

Appendix A

Appendix

A.1 appendix A

The following is the IEEE69 test system used in simulations.

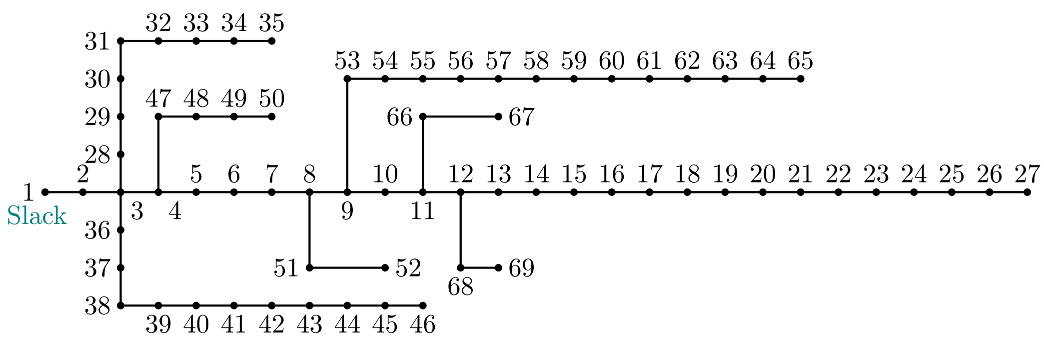

Figure A.1: IEEE69 test system

Table A.1: Single-phase equivalent of the IEEE69 test system

From	**To**	**R**	**X**	**B**	**P**	**Q**	α
1	2	3,11667E-06	0,0000075	0	0	0	0
2	3	3,11667E-06	0,0000075	0	0	0	0
3	4	9,35001E-06	0,0000224	0	0	0	0
4	5	0,000156457	0,0001833	0	0	0	0

Continued on next page

Table A.1 – continued from previous page

From	To	R	X	B	P	Q	α
5	6	0,002281404	0,0011619	0	0,0026	0,0022	0
6	7	0,002374904	0,0012099	0	0,0404	0,03	1
7	8	0,000574714	0,0002930	0	0,075	0,054	2
8	9	0,000307304	0,0001565	0	0,03	0,022	0
9	10	0,005105108	0,0016874	0	0,028	0,019	2
10	11	0,001166882	0,0003858	0	0,145	0,104	2
11	12	0,0044344	0,0014655	0	0,145	0,104	2
12	13	0,006420343	0,0021193	0	0,008	0,005	1
13	14	0,00650761	0,0021505	0	0,008	0,0055	1
14	15	0,006594877	0,0021792	0	0	0	0
15	16	0,001225475	0,0004052	0	0,0455	0,03	0
16	17	0,002333764	0,0007717	0	0,06	0,035	2
17	18	4,61267E-05	0,0000100	0	0,06	0,035	0
18	19	0,002042043	0,0006751	0	0	0	0
19	20	0,001312742	0,0004301	0	0,001	0,0006	1
20	21	0,00212931	0,0007037	0	0,114	0,081	2
21	22	8,72668E-05	0,0000287	0	0,005	0,0035	1
22	23	0,000991725	0,0003279	0	0	0	0
23	24	0,002158607	0,0007137	0	0,028	0,02	0
24	25	0,004667527	0,0015428	0	0	0	0
25	26	0,00192548	0,0006364	0	0,014	0,01	0
26	27	0,001079615	0,0003565	0	0,014	0,01	1
3	28	2,74267E-05	0,0000673	0	0,026	0,0186	2
28	29	0,000398934	0,0009755	0	0,026	0,0186	1
29	30	0,002479624	0,0008197	0	0	0	0
30	31	0,000437581	0,0001446	0	0	0	0
31	32	0,002187903	0,0007231	0	0	0	0
32	33	0,005229775	0,0017553	0	0,014	0,01	1
33	34	0,01064655	0,0035193	0	0,0195	0,014	1
34	35	0,009187948	0,0030375	0	0,006	0,004	1
3	36	2,74267E-05	0,0000067	0	0,026	0,01855	1
36	37	0,000398934	0,0009755	0	0,026	0,01855	2
37	38	0,000656371	0,0007667	0	0	0	0
38	39	0,000189494	0,0002213	0	0,024	0,017	1
39	40	1,122E-05	0,0000131	0	0,024	0,017	1

Continued on next page

Table A.1 – continued from previous page

From	To	R	X	B	P	Q	α
40	41	0,004539744	0,0053040	0	0,0012	0,001	1
41	42	0,001932336	0,0022583	0	0	0	0
42	43	0,000255567	0,0002980	0	0,006	0,0043	2
43	44	5,73468E-05	0,0000723	0	0	0	0
44	45	0,000678811	0,0008558	0	0,03922	0,0263	1
45	46	5,61001E-06	0,0000075	0	0,03922	0,0263	2
4	47	2,11934E-05	0,0000524	0	0	0	0
47	48	0,000530457	0,0012984	0	0,079	0,0564	1
48	49	0,001806423	0,0044201	0	0,3847	0,2745	1
49	50	0,000512381	0,0012535	0	0,3847	0,2745	1
8	51	0,000578454	0,0002948	0	0,0405	0,0283	1
51	52	0,002068847	0,0006944	0	0,0036	0,0027	2
9	53	0,001084602	0,0005523	0	0,00435	0,0035	0
53	54	0,001265369	0,0006445	0	0,0264	0,019	0
54	55	0,001771516	0,0009020	0	0,024	0,0172	0
55	56	0,001753439	0,0008932	0	0	0	0
56	57	0,009911016	0,0033267	0	0	0	0
57	58	0,004885071	0,0016394	0	0	0	0
58	59	0,001896183	0,0006271	0	0,1	0,072	1
59	60	0,002406694	0,0007305	0	0	0	0
60	61	0,003163422	0,0016113	0	1,244	0,888	0
61	62	0,000607128	0,0003092	0	0,032	0,023	2
62	63	0,000903835	0,0004600	0	0	0	0
63	64	0,00442879	0,0022558	0	0,227	0,162	0
64	65	0,00648891	0,0033049	0	0,059	0,042	1
11	66	0,001254149	0,0003809	0	0,018	0,013	1
66	67	2,92967E-05	0,0000087	0	0,018	0,013	1
12	68	0,004608934	0,0015234	0	0,028	0,02	1
68	69	2,92967E-05	0,0000100	0	0,028	0,02	2

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